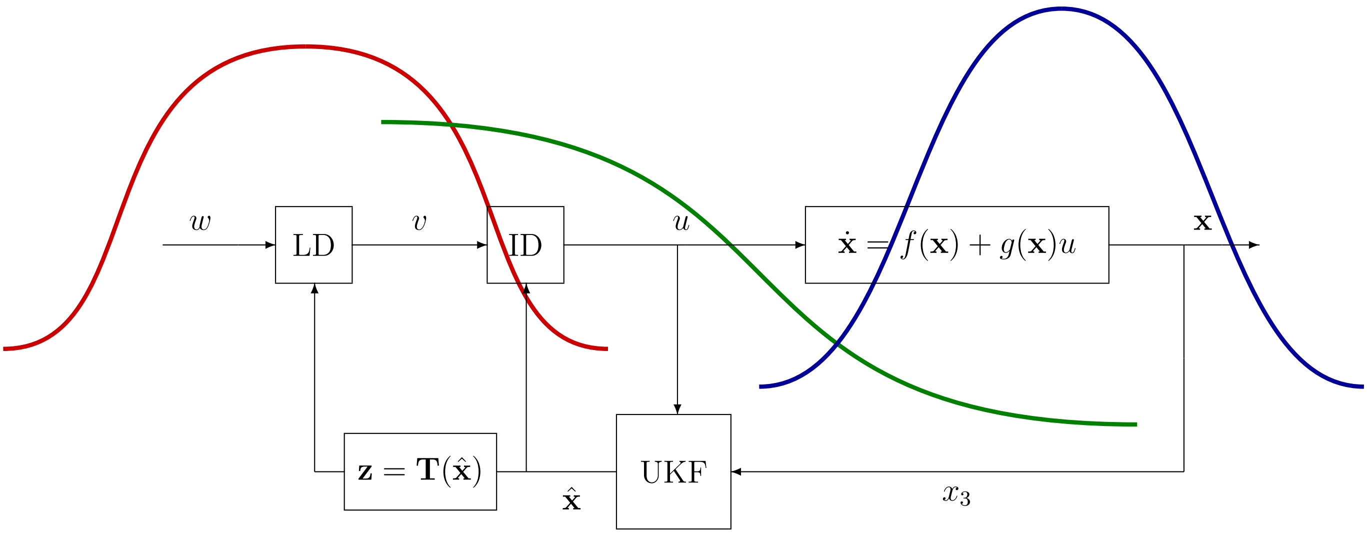

Modelling and Compensation of Nonlinear Loudspeaker

Master's Thesis in Systems, Control and Mechatronics

DAVID JAKOBSSON
MARCUS LARSSON

Department of Signals and Systems
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2010

Master's Thesis EX033/2010

MASTER'S THESIS EX033/2010

Modelling and Compensation of Nonlinear Loudspeaker

Master's Thesis in Systems, Control and Mechatronics

DAVID JAKOBSSON
MARCUS LARSSON

Department of Signals and Systems
CHALMERS UNIVERSITY OF TECHNOLOGY

Göteborg, Sweden 2010

Modelling and Compensation of Nonlinear Loudspeaker
DAVID JAKOBSSON
MARCUS LARSSON

©DAVID JAKOBSSON, MARCUS LARSSON, 2010

Master's Thesis EX033/2010
Department of Signals and Systems
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

Cover:
A diagram of the observer based system with the three major nonlinear functions in background.

Göteborg, Sweden 2010

Modelling and Compensation of Nonlinear Loudspeaker
Master's Thesis in Systems, Control and Mechatronics
DAVID JAKOBSSON
MARCUS LARSSON
Department of Signals and Systems
Chalmers University of Technology

Abstract

The nonlinear response of the loudspeaker is a undesired phenomenon which produces audible distortion. The construction of the loudspeaker is a trade-off between various factors and therefore makes it difficult to build a linear loudspeaker.

This work takes another approach to address this issue. By using digital signal processing, the terminal voltage can be pre-distorted to compensate for the nonlinear behavior and render the system input-output linear.

In order to compute a controller which achieves this, a model of the moving coil loudspeaker was implemented. This model was made from the extended Thiele-Small model. The theory of exact input-output linearization was used to compute the control law from this model.

Since the control law assumes full-state feedback, a state estimator is used. Two types of state estimators, a feed-forward based and a observer based, were computed and analyzed. The feed-forward one simply uses the terminal voltage to estimate the states of the loudspeaker and can therefore be implemented without sensors. The observer one uses the theory of the unscented Kalman filter to estimate the states using the terminal voltage and the current passing through the loudspeaker.

The simulation results showed that the loudspeaker can be rendered fully linear assuming that the hardware does not limit the control signal and that the loudspeaker model is perfect. Various simulation were done to simulate hardware limitation and process noise. This resulted as expected, that the system was able to compensate for the nonlinear behavior but its performance was still affected by the limitation and the noise. The unscented Kalman filter proved to be more capable of estimating the states than the feed-forward estimator when affected by process noise.

Keywords: Nonlinear loudspeaker, Active loudspeaker, Compensation, Input-output linearization, Modelling, Feed-forward, Unscented Kalman filter

Contents

Glossary

Loudspeaker	
u	Voltage at loudspeaker terminals
$\mathbf{x}$	State vector
x_1	First state - Cone displacement (x)
x_2	Second state - Cone velocity ($\dot{x}$)
x_3	Third state - Terminal current (i)
x_4	Forth state - Current through L_2 (i_2)
x	Cone displacement
$\dot{x}$	Cone velocity
i	Terminal current
i_2	Current flowing through L_2
R_e	Voice coil resistance (DC)
L_e	Voice coil inductance
L_2	Parainductance
R_2	Eddy current resistance
Bl	Force factor
F_m	Reluctance force
C_{ms}	Suspension compliance
R_{ms}	Suspension mechanical resistance
M_{ms}	Diaphragm mechanical mass
M	Diaphragm mechanical mass + air load
T_v	Voice coil temperature

Controller	
w	Signal source
v	Linear dynamics control signal
u	Inverse dynamics control signal (Terminal voltage)
LD	Linear dynamics
ID	Inverse dynamics
$\hat{\mathbf{x}}$	Predicted state vector
$\mathbf{z}$	Transformed state vector

Observer	
UKF	Unscented Kalman filter
$\mathbf{m}$	State mean
$\mathbf{P}$	State covariance
$\mathbf{Q}$	Covariance of the process noise
$\mathbf{R}$	Covariance of the measurement noise
$\mathbf{X}$	Sigma points
$\boldsymbol{\mu}$	Predicted mean
$\mathbf{S}$	Measurement covariance
$\mathbf{C}$	Cross-covariance
$\mathbf{K}$	Filter gain

Distortion	
HD	Harmonic distortion
THD	Total harmonic distortion
IMD	Intermodulation distortion

Preface

In this thesis the nonlinear behavior of the moving coil loudspeaker has been studied and a control law which achieves exact input-output linearization has been derived. As full state-feedback is not applicable for the purpose of this system and as such, a observer has been made which estimates the states based on the input to the loudspeaker and the terminal current.

This work has been carried out for Actiwave AB in the spring of 2010 at the Department of Signals and Systems, Chalmers University of Technology, Sweden. Supervisors for this work were Prof. Tomas McKelvey at the Department of Signals and Systems, Chalmers University of Technology and Pär Gunnars Risberg at Actiwave AB.

Aknowledgements

First of all, we would like to say our thanks to our supervisors, Prof. Tomas McKelvey for his valuable time and technical knowledge and Pär Gunnars Risberg for his support. Also, we would like to thank Börje Wijk at the Department of Applied Acoustics for his lab support.

Göteborg Juni 2010
David Jakobsson, Marcus Larsson

1 Introduction

Since the invention of the moving coil loudspeaker in the 19th century, its principle physical properties have remained the same. Improvements has been made but most often in types of material usages. That development has led to that loudspeakers in the high fidelity range has a delicate construction process and costly materials.

In recent years, more momentum has been put into understanding the dynamics of the moving coil loudspeaker which is somewhat of a difficult task since it can be classified as a nonlinear time-variant system [Kli06]. With that knowledge, the ways to improve the loudspeaker has become clearer but still leave one to choose between, e.g. fidelity, safety, size and expenses. This challenge has led to the more recent idea of the active loudspeaker, i.e. using digital signal processing to compensate for the loudspeakers short-comings.

One of those short-comings is that the loudspeaker is a nonlinear system which in turn produces unwanted distortion. This thesis will address this problem using digital signal processing to compensate this nonlinear behavior.

1.1 Purpose

Loudspeakers behave differently in the small and large signal domain. This is due to the nonlinear behavior of the loudspeaker which it inherits from its amplitude dependency. The loudspeaker has therefore stronger nonlinear behavior when it is playing at higher amplitudes. This nonlinear behavior does depend on the input stimulus, e.g. frequency, amplitude and phase and should thus be understood as symptoms rather than a complete descriptions of the large signal performance. Those symptoms can be seen as a integer multiples of the fundamental frequency and are called harmonic (HD) and intermodulation distortion (IMD). [Kli06]

As the human perception of sound waves is acknowledged as a nonlinear system, one might wonder if a linear acoustic source is desired or necessary. Extensive studies can be found regarding this subject, i.e. how humans perceive sound waves. The studies shows that harmonic distortion does not sound all that bad. On the other hand, the harmonic distortion is also the source of intermodulation distortion which is perceived as an unpleasant artifact. [BNS+98]

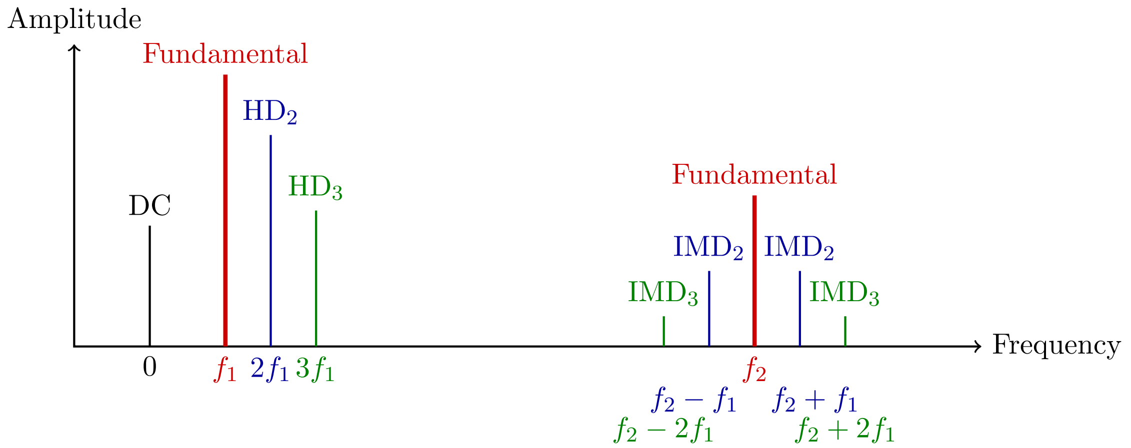

Figure 1.1: A frequency spectrum demonstrating HD and IMD.

In Figure 1.1 it can be seen how the harmonic distortion for two tones affect each other, which results in intermodulation distortion. Note that this is a simplification of the reality and usually there are more than two harmonics distortion components present.

The challenge, to compensate for this distortion, is to achieve input-output linear relationship between the loudspeaker cone movement and the input stimulus. Since a consumer product is the ultimate goal, a system that achieves this need to be cheap and free of user interaction. As this is the case, as much as possible need to be done using models and feed-forward techniques and limit the use of expensive sensors.

1.2 Limitations

A few limitations was needed to fit the project to the time schedule. Firstly, it is known that the loudspeaker is a time-varying system, which time dependency comes mostly from the temperature of the voice coil. This is due to the fact that the loudspeaker has a particular low efficiency, around 5%, where the rest of the energy will be dissipated as heat. The other time-varying effect is due to aging and is a much slower process. This work will not address these issues but will describe a few measures in the Chapter 7 to overcome them. [Kli04]

This work will also mostly be limited to a simulation model to verify the observer described later on. This is due to the process of the implementing a feedback loop in a digital signal processor (DSP). The feed-forward based controller will though be tested which will allow for verification of the control law together with the loudspeaker model.

This report will mostly address the loudspeaker model solely on the driver, or in other words without enclosure, but since the closed box system is very similar to it, the report will describe it shortly. In Chapter 3.1, it will be discussed briefly how the model needs to be extended to include vented boxes.

1.3 Approach

The approach which was taken was to model the loudspeaker as a state-space system in Simulink. The state-space model was made using extended Thiele - Small lumped parameter model of the loudspeaker. This results in four states defined as the vector $\mathbf{x}$ which are the following:

- $x_1 = x$ - The displacement of the cone
- $x_2 = \dot{x}$ - The velocity of the cone
- $x_3 = i$ - The current passing through the loudspeaker
- $x_4 = i_2$ - The current passing through the parainductance (L_2) of the voice coil.

One should note the fundamental difference between the state vector $\mathbf{x}$ and the cone displacement x. This notation will be used throughout the report and should not be confused.

With the loudspeaker model in place, an exact input-output control law was derived and applied. The control law is divided into two parts, the ID (Inverse dynamics) controller and the LD (Linear dynamics) controller. The ID controller sees to that an exact input-output linear behavior applies between its input and the displacement of the loudspeaker cone and the systems linear dynamics can be tuned using the LD controller. This controller requires a full-state feedback which need to be updated every time step. For this task, two approaches were made, a feed-forward system and a observer based system.

The feed-forward system calculates the state variables by solely knowing the input u which should make the system quite vulnerable to process noise but has the advantages of simplicity and a sensor-free set-up.

The observer based feed-forward system takes another approach. By measuring the current i and knowing the input voltage u it can estimate the other three state variables and allow for the controller to operate smoothly. The observer which was chosen to estimate the state variables was the unscented Kalman filter. Block diagram of the observer based system is shown below.

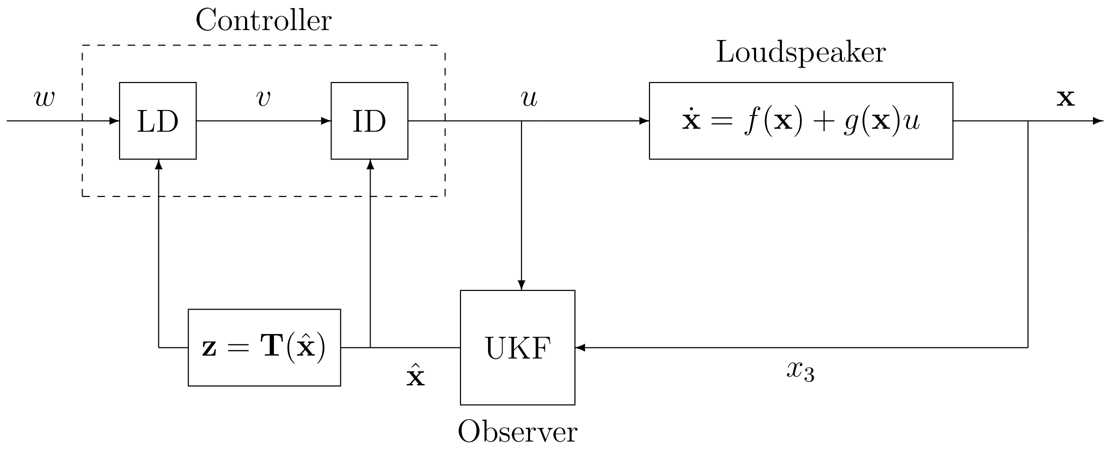

Figure 1.2: A diagram of the observer based system.

2 Theory

This chapter aims to give the reader the needed knowledge to follow the methodology in the upcoming chapters. It starts with an explanation of the basic principles of the moving coil loudspeaker. This is followed by control theory regarding exact input-output linearization and the last chapter is dedicated to the unscented Kalman filter.

2.1 Moving coil loudspeaker

A loudspeaker is a device used to convert a electric signal into acoustic waves. The moving coil loudspeaker is, by far, the most common version. Figure 2.1 shows its cross section with the consisting parts marked out.

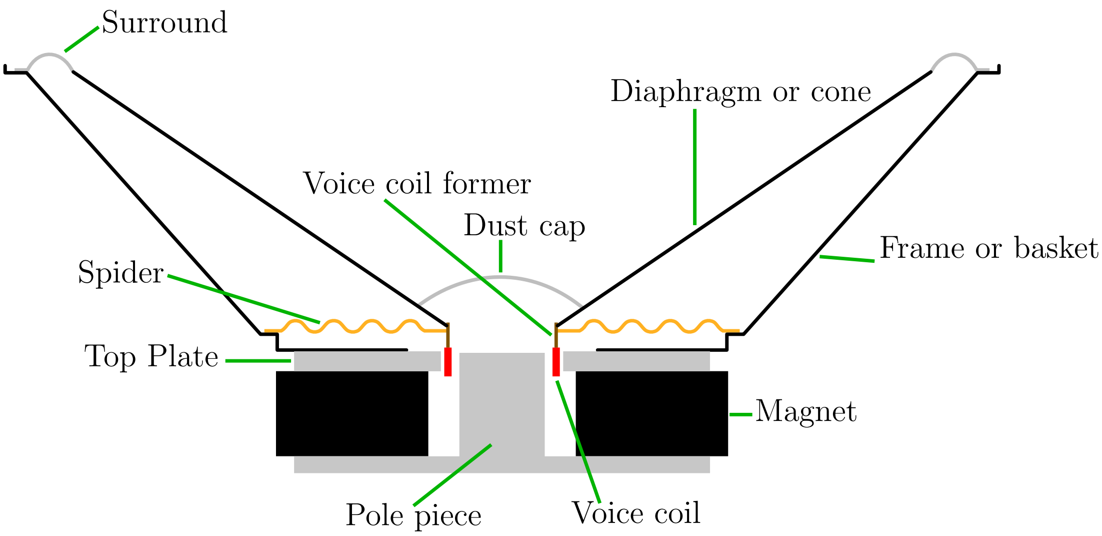

Figure 2.1: Cross section of the moving coil loudspeaker [Iai10].

The sound is created when the diaphragm moves. As can be seen, the diaphragm is fixed to the surround in one end and to the spider and the voice coil former in the other end. The function of the surround and the spider is to keep the diaphragm and the voice coil centered in the frame as well as generating a restoring force which moves the voice coil back into rest position. The voice coil is what sets the diaphragm into movement which is done by applying an alternating current to the voice coil. Since it is positioned in a permanent magnetic field, this will result in a force moving the voice coil and its former as well as the diaphragm. [BX08]

The small signal behavior of a loudspeaker can be accurately modeled using linear theory. For higher amplitudes, this is not the case, as all loudspeakers have nonlinearities generating distortion not present in the input. The nonlinear behavior is a consequence of the properties of the materials used in various parts of the loudspeaker. When constructing a loudspeaker, these properties are a trade-off between e.g. cost, size, linear behavior and sensitivity. [Kli06]

Below follows a short description of the major nonlinearities for low frequencies, as well as the basic physical reasons for their presence.

Force factor $Bl(x)$

The electromagnetic conversion from electrical to mechanic energy is described by the force factor $Bl(x)$. Here B is the magnetic flux density in the gap and l is the effective length of the voice coil wire in the gap. Obviously the force factor decreases when the voice coil leaves the gap which makes the force factor displacement dependent.

Voice coil inductance $L_e(x)$, L_2 & R_2

The inductance of the voice coil is also dependent on the voice coil displacement. This is due to the surrounding material, which is air for the part of the voice coil outside the gap and steel (which decreases magnetic resistance) for the part of the coil inside the gap. The current in the voice coil is also causing a magnetic field penetrating the magnet and iron introducing eddy currents which causes additional losses.These losses are increasing with frequency and making a significant impact from approximately 200 Hz depending on the loudspeaker used [RLRV10].

Suspension compliance $C_{ms}(x)$

The suspension parts are usually made of rubber, impregnated fabric or polymer. The suspension behavior is similar to a normal linear spring at small excursions but increases quicker at larger excursions causing a nonlinear force dependent on the voice coil displacement.

In addition to these three, there exists more nonlinearities affecting the output of a loudspeaker such as the Doppler effect and break-up modes in the material which are only in effect at higher frequencies. Additionally to those, there are also the port nonlinearity for vented systems and material defects causing rub and buzz. Since this report is focused on low frequency distortion for closed box loudspeakers and solo drivers, these nonlinearities will not be described here but more information can be found, e.g. in [Kli06], [BH09] or [Ped08].

2.2 Exact input-output linearization

Exact input-output linearization is a technique to transform a nonlinear SISO system into a linear one by the means of a control law. This is done in two steps, firstly the nonlinear system is transformed into a integrator decoupled system by the means of a state feedback. Secondly, also by the means of state feedback the appropriate linear dynamics are chosen with common linear control theory methods. [SSH96]

Consider the nonlinear SISO system given by the state-space representation

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \\ y &= \mathbf{h}(\mathbf{x}) \end{aligned} \tag{2.1}$$

where u is the input and y is the output. Exact input-output linearization refers to that a control law is generated which achieves a linear differential relation between the output y and a new input v. This is significantly different from and should not be confused with "approximate" linearization where a linear approximation is found to a nonlinear function at a given point. [Isi95]

The goal of the exact input-output technique is to transform the system with $\mathbf{z} = \mathbf{T}(\mathbf{x})$ into normal form which can be seen as a feedback law described below.

$$u = \boldsymbol{\alpha}(\mathbf{x}) + \boldsymbol{\beta}(\mathbf{x})v \tag{2.2}$$

To explain this theory one must first understand the notion of Lie derivative and relative degree [SSH96]. Considering the Lie derivative of $\mathbf{h}(\mathbf{x})$ along $\mathbf{f}(\mathbf{x})$ is defined as

$$\mathbf{L}_f\mathbf{h}(\mathbf{x}) = \frac{\mathbf{dh}(\mathbf{x})}{\mathbf{dx}}\mathbf{f}(\mathbf{x}) \tag{2.3}$$

Likewise, the Lie derivative of $\mathbf{h}(\mathbf{x})$ along $\mathbf{g}(\mathbf{x})$ is

$$\mathbf{L}_g\mathbf{h}(\mathbf{x}) = \frac{\mathbf{dh}(\mathbf{x})}{\mathbf{dx}}\mathbf{g}(\mathbf{x}) \tag{2.4}$$

The time derivative of y as is described in equation 2.1 can be expressed with Lie derivative notions as

$$\begin{aligned}\dot{\mathbf{y}} &= \frac{\mathbf{dh}(\mathbf{x})}{\mathbf{dx}}\dot{\mathbf{x}} \\ &= \frac{\mathbf{dh}(\mathbf{x})}{\mathbf{dx}}\mathbf{f}(\mathbf{x}) + \frac{\mathbf{dh}(\mathbf{x})}{\mathbf{dx}}\mathbf{g}(\mathbf{x})u \\ &= \mathbf{L}_f\mathbf{h}(\mathbf{x}) + \mathbf{L}_\mathbf{g}\mathbf{h}(\mathbf{x})u\end{aligned} \tag{2.5}$$

Knowing that, the linearizing diffeomophism or as described earlier, the transformation coordination can be put forth as follows

$$\mathbf{z} = \mathbf{T}(\mathbf{x}) = \begin{bmatrix} \mathbf{h}(\mathbf{x}) \\ \mathbf{L}_f\mathbf{h}(\mathbf{x}) + \mathbf{L}_g\mathbf{h}(\mathbf{x})u \\ \mathbf{L}_f^2\mathbf{h}(\mathbf{x}) + \mathbf{L}_g\mathbf{L}_f\mathbf{h}(\mathbf{x})u \\ \vdots \\ \mathbf{L}_f^n\mathbf{h}(\mathbf{x}) + \mathbf{L}_g\mathbf{L}_f^{n-1}\mathbf{h}(\mathbf{x})u \\ \Psi_1 \\ \vdots \\ \Psi_{n-r} \end{bmatrix} \tag{2.6}$$

The relative degree of the system is the smallest derivative of the function $\mathbf{h}(\mathbf{x})$ which explicitly depends on the input u [SL91]. If the relative degree is smaller than the number of states, Ψ can be chosen so that equation 2.7 holds [SL91].

$$\mathbf{L}_\mathbf{g}\Psi = 0 \tag{2.7}$$

In case of a well defined relative degree, a control law which achieves a linear differential relation between the output y and the input v can be calculated as

$$\begin{aligned}u &= -\frac{\mathbf{L}_f^n\mathbf{h}(\mathbf{x})}{\mathbf{L}_g\mathbf{L}_f^{n-1}\mathbf{h}(\mathbf{x})} + \frac{1}{\mathbf{L}_g\mathbf{L}_f^{n-1}\mathbf{h}(\mathbf{x})}v \\ &= \boldsymbol{\alpha}(\mathbf{x}) + \boldsymbol{\beta}(\mathbf{x})v\end{aligned} \tag{2.8}$$

This is called the inverse dynamics, since it cancels both the linear and the nonlinear dynamics of the system.

If the relative degree of the system is the same as the number of states, one has what is called exact state linearization. This is an important case since this eliminate any zero dynamics in the system. Zero dynamics are nonlinear dynamics affecting the state variables which is not visible in the output. It is still possible to achieve feedback linearization even though the relative degree is smaller than the number of state variables. One has to be aware though, that the zero dynamics can be unstable and if left to grow unbound, it can be harmful to the system. In practice the zero dynamics can be, if not stable, at least controllable and therefore allows for the controller to work properly. [GL00]

2.3 Unscented Kalman filter

The UKF is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation [JU04].

The unscented Kalman filter (UKF) is a novel idea to estimate the state variables of a nonlinear system by calculating the mean. It belongs to a bigger class of filters called Sigma-Point Kalman filters which make use of statistical linearization techniques. [Ter10]

It draws it's name from the unscented transform which is a method for statistically calculating a stochastic variable which goes through a nonlinear transformation. The non-augmented UKF, which assumes additive noise, uses the unscented transformation to make a Gaussian approximation to the nonlinear problem given as [Ter10]

$$\begin{aligned} \mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1}, k-1) + \mathbf{q}_{k-1} \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, k) + \mathbf{r}_k \end{aligned} \tag{2.9}$$

where $\mathbf{x}_k$ is the state vector, $\mathbf{y}_k$ is the measurement vector, $\mathbf{q}_{k-1}$ is the process noise and $\mathbf{r}_k$ is measurement noise defined as

$$\begin{aligned} \mathbf{x}_k &\in \mathbb{R}^n \\ \mathbf{y}_k &\in \mathbb{R}^m \\ \mathbf{q}_{k-1} &\sim N(\mathbf{0}, \mathbf{Q}_{k-1}) \\ \mathbf{r}_k &\sim N(\mathbf{0}, \mathbf{R}_k) \end{aligned} \tag{2.10}$$

Like the Kalman filter, the UKF consists of two steps, prediction and update. Unlike the Kalman filter though, the UKF makes use of so called sigma points which is used to better capture the distribution of $\mathbf{x}$. The mean values of that distribution will here be indicated as $\mathbf{m}$. The sigma points $\mathbf{X}$ are then propagated through the nonlinear function $\mathbf{f}$ and the moments of the transformed variable estimated. [JS10]

For the non-augmented UKF a set of $2n+1$ of sigma points is used, where n is the order of the states. Before going through the prediction and update steps the associated weight matrices $\mathbf{W}_m$ and $\mathbf{W}_c$ need to be defined. This is done as follows

$$\begin{aligned} W_m^{(0)} &= \lambda/(n+\lambda) \\ W_c^{(0)} &= \lambda/(n+\lambda) + (1 - \alpha^2 + \beta) \\ W_m^{(i)} &= 1/\{2(n+\lambda)\}, \quad i = 1, \cdots, 2n \\ W_c^{(i)} &= 1/\{2(n+\lambda)\}, \quad i = 1, \cdots, 2n \end{aligned} \tag{2.11}$$

where $W_m^{(0)} \cdots W_m^{(i)}$ and $W_c^{(0)} \cdots W_c^{(i)}$ are column vectors for the weight matrices. The scaling parameter λ is defined as

$$\lambda = \alpha^2(n+\kappa) - n \tag{2.12}$$

where α, β and κ are positive constants which can be used to tune the UKF by modifying its weight matrices. The prediction and update steps can now be computed as follows:

- **Prediction**
  The prediction step computes the predicted state mean $\mathbf{m}_k^-$ and the predicted covariance $\mathbf{P}_k^-$ by calculating the sigma points $\mathbf{X}_{k-1}$

$$
\begin{aligned}
\mathbf{X}_{k-1} &= \begin{bmatrix}\mathbf{m}_{k-1} & \cdots & \mathbf{m}_{k-1}\end{bmatrix} + \sqrt{c}\begin{bmatrix}\mathbf{0} & \sqrt{\mathbf{P}_{k-1}} & -\sqrt{\mathbf{P}_{k-1}}\end{bmatrix} \\
\hat{\mathbf{X}}_k &= \mathbf{f}(\mathbf{X}_{k-1}, k-1) \\
\mathbf{m}_k^- &= \mathbf{X}_k \mathbf{W}_m \\
\mathbf{P}_k^- &= \hat{\mathbf{X}}_k \mathbf{W}_c [\hat{\mathbf{X}}_k]^T + \mathbf{Q}_{k-1}
\end{aligned} \tag{2.13}
$$

- **Update**
  The update step computes the predicted mean $\boldsymbol{\mu}_k$, measurement covariance $\mathbf{S}_k$ and the measurement and state cross-covariance $\mathbf{C}_k$

$$
\begin{aligned}
\mathbf{X}_k^- &= \begin{bmatrix}\mathbf{m}_k^- & \cdots & \mathbf{m}_k^-\end{bmatrix} + \sqrt{c}\begin{bmatrix}\mathbf{0} & \sqrt{\mathbf{P}_k^-} & -\sqrt{\mathbf{P}_k^-}\end{bmatrix} \\
\mathbf{Y}_k^- &= \mathbf{h}(\mathbf{X}_k^-, k) \\
\mu_k^- &= \mathbf{Y}_k^- \mathbf{W}_m \\
\mathbf{S}_k &= \mathbf{Y}_k^- \mathbf{W}_c [\mathbf{Y}_k^-]^T + \mathbf{R}_k \\
\mathbf{C}_k &= \mathbf{X}_k^- \mathbf{W}_c [\mathbf{Y}_k^-]^T
\end{aligned} \tag{2.14}
$$

  At last the filter gain $\mathbf{K}_k$, the updated state mean $\mathbf{m}_k$ and the covariance $\mathbf{P}_k$ are computed

$$
\begin{aligned}
\mathbf{K}_k &= \mathbf{C}_k \mathbf{S}_k^{-1} \\
\mathbf{m}_k &= \mathbf{m}_k^- + \mathbf{K}_k [y_k - \boldsymbol{\mu}_k] \\
\mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T
\end{aligned} \tag{2.15}
$$

As one could guess, initial values for the mean $\mathbf{m}$ and the covariance $\mathbf{P}$ need to be chosen for the first run. Afterwards, the algorithm can simply be run iteratively.

3 Modelling and compensation

This chapter aims to give an understanding of how a model is made of the moving coil loudspeaker as well as explaining how the exact input-output linearization and state estimation techniques was utilized to compensate for the distortion in the model.

This chapter is divided into the three basic components of the simulated system, namely

- The moving coil loudspeaker model
- The controller
- The state estimation

The moving coil loudspeaker model chapter explains how the model is derived from its electrical equivalent circuit and then transfered into state-space form. It also explains the nonlinear components of the model and how they vary with the cone displacement. The controller chapter describes how the exact input-output theory is applied to the model and how the performance of the controller can be tuned by varying some of its parameters. The state estimation chapter explains how two different measures were taken to estimate the current state of the loudspeaker. This includes the feed-forward technique and the application of the unscented Kalman filter.

3.1 Moving coil loudspeaker model

A popular way to model loudspeakers is to use an electrical equivalent circuit. This model and its parameters are named Thiele-Small after the two Australians who pioneered this theory in the 1970's [Thi71]. Those parameters are more or less industry standard nowadays and are usually specified by transducer manufacturers.

More recently it has been accepted that the loudspeaker is a nonlinear system, where its major nonlinearties depend on the cone displacement. In this chapter an explanations will be given of how the loudspeaker model is derived from the extended Thiele-Small model and how the major nonlinearties are modelled.

3.1.1 Model derivations

The original Thiele-Small model models the low frequency performance of the moving coil loudspeaker. It is defined as a linear one which is sufficient to predict the small signal behavior of the loudspeaker. In other words it is limited to when the cone displacement is small. To use this model to predict the large signal behavior some changes has to be made to include the nonlinear dynamics. An extended version of the Thiele-Small model where the nonlinear dynamics have been taken into account, can be seen in Figure 3.1.

This circuit is only equivalent for the solo driver but to extend the model to include a closed box is quite simple as the only parameter that needs to be changed is the mass. For vented boxes this model needs to be extended to include the nonlinear dynamics of air flow.

A LR-2 model was used to describe the eddy currents occurring at higher frequencies. This can be seen in the circuit as the functions $R_2(x)$ and $L_2(x)$. This was chosen as it can be easily implemented in the circuit and will therefore not cause problems later on when synthesizing the controller. [DKOB04]

The dynamics of this circuit can more easily be described by dividing it into two parts, the electrical and mechanical part. This can be seen in Figure 3.1 as the left side of the converter $Bl(x)$ and the right side respectively. The differential equations which describe

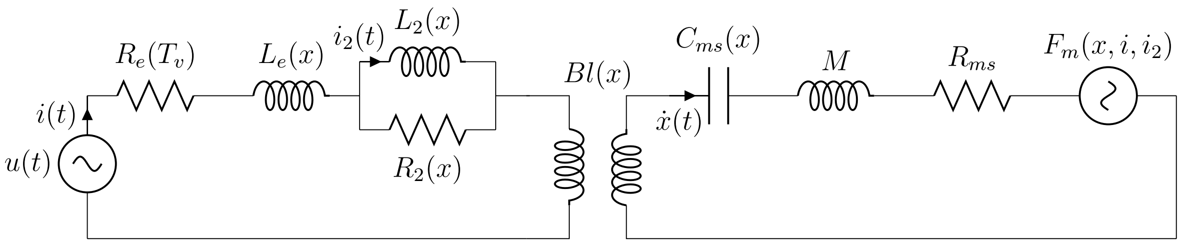

Figure 3.1: The equivalent circuit of the moving coil loudspeaker.

the electrical part are described by equation 3.1 and 3.2, while equation 3.3 describes the mechanical parts and equation 3.4 describes the reluctance force. [BH09]

$$u(t) = i(t)R_e(T_v) + \frac{d(L_e(x)i(t))}{dt} + \frac{d(L_2(x)i_2(t))}{dt} + Bl(x)\frac{dx}{dt} \tag{3.1}$$

$$\frac{d(L_2(x)i_2(t))}{dt} = (i(t) - i_2(t))R_2(x) \tag{3.2}$$

$$Bl(x)i(t) - F_m(x,i,i_2) = M\frac{d^2x}{dt^2} + R_{ms}\frac{dx}{dt} + \frac{x}{C_{ms}(x)} \tag{3.3}$$

$$F_m(x,i,i_2) \approx -\frac{i^2(t)}{2}\frac{dL_e(x)}{dx} - \frac{i_2^2(t)}{2}\frac{dL_2(x)}{dx} \tag{3.4}$$

The reluctance force $F_m(x,i,i_2)$ as can be seen above, is here modelled as an approximation. It drives the mechanical system directly and creates distortion in the full audio band. Early on, it was a major driving-force of the moving coil loudspeaker but currently it is considered an undesired effect which is kept as low as possible. [Kli06]

As the goal is to describe the model on a state-space form, those equations can be solved for the derivatives of the states as seen below.

$$\frac{d^2x}{dt^2} = \frac{1}{M}\left(-\frac{x(t)}{C_{ms}(x)} - R_{ms}\frac{dx(t)}{dt} + i(t)\left(Bl(x) + \frac{1}{2}\frac{dL_e(x)}{dx}i(t)\right) + \frac{1}{2}\frac{dL_2(x)}{dx}i_2^2\right) \tag{3.5}$$

$$\frac{di}{dt} = \frac{1}{L_e(x)}\left(-\frac{dx}{dt}\left(Bl(x) + \frac{dL_e}{dx}i(t)\right) - i(t)\left(R_e(T_v) + R_2(x)\right) + i_2R_2(x) + u(t)\right) \tag{3.6}$$

$$\frac{di_2}{dt} = \frac{1}{L_2(x)}\left(i(t)R_2(x) - i_2\left(R_2(x) + \frac{dL_2(x)}{dx}\frac{dx}{dt}\right)\right) \tag{3.7}$$

This gives that the choice of the states $\mathbf{x}$ is as follows

$$\mathbf{x} = \begin{bmatrix} x(t) & \dot{x}(t) & i(t) & i_2(t) \end{bmatrix}^T \tag{3.8}$$

Using equations 3.5 - 3.8 the system can be described with a nonlinear state space equation of the form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \\ y &= \mathbf{h}(\mathbf{x}) \end{aligned} \tag{3.9}$$

where $y = \mathbf{h}(\mathbf{x})$ specifies the output, which in this case is the cone displacement x_1. Even though equation 3.9 is nonlinear, it will be displayed here on the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \tag{3.10}$$

for convenience. Note though that the states $\mathbf{x}$ do appear in the matrices $\mathbf{A}$ and $\mathbf{b}$. The resulting loudspeaker model can be seen below.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-1}{MC_{ms}(x)} & \dfrac{-R_{ms}}{M} & \dfrac{Bl(x) + \dfrac{1}{2}\dfrac{dL_e(x)}{dx}x_3}{M} & \dfrac{\dfrac{1}{2}\dfrac{dL_2(x)}{dx}x_4}{M} \\ 0 & \dfrac{-Bl(x) - \dfrac{dL_e(x)}{dx}x_3}{L_e(x)} & \dfrac{-R_e(T_v) - R_2(x)}{L_e(x)} & \dfrac{R_2(x)}{L_e(x)} \\ 0 & 0 & \dfrac{R_2(x)}{L_2(x)} & \dfrac{-R_2(x) - \dfrac{dL_2(x)}{dx}x_2}{L_2(x)} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L_e(x)} \\ 0 \end{bmatrix} u \tag{3.11}$$

3.1.2 Loudspeaker nonlinearities

It is generally accepted that the major nonlinearities of the loudspeaker is the force factor Bl, the voice coil inductance L_e and the compliance of the suspension C_{ms}. There exists other nonlinearties as well, which include the eddy current parameters R_2 and L_2 and DC resistance of the voice coil which changes with the temperature T_v. The last one is though considered slowly varying compared to the other which vary with the displacement of the cone x_1. The eddy current parameters R_2 and L_2 are also considered weak nonlinearities in a loudspeaker without shorting-rings [DKOB04].

In order to simplify the model for the simulation, a few approximations have been made. Although those approximations where used in the simulation, no approximations where used when modelling the loudspeaker, synthesizing the controller or the state estimation. In other words, some of the nonlinear functions seen in the entire system were approximated as linear or constant in the simulation but are given as nonlinear in the equation. This was done so that the model would be valid even if one would choose to explicitly define the approximated functions as nonlinear.

The functions which were approximated were the eddy current parameters R_2 and L_2 and the temperature dependent DC resistance R_e. Since the eddy current parameters are weak nonlinearities they were here considered as constants as it was hard to get an estimation of the nonlinear functions. Thermal effects in the loudspeaker are considered significant for effective compensation of the its nonlinearities [Kli04]. Although interesting factor, it was not thought to fit into the time scope of the project to model those temperature dependencies. In the simulation the DC resistance R_e is therefore considered as constant. The three major nonlinearities Bl, L_e and C_{ms} are considered to have temperature dependency as well.

The popular way of modelling the major nonlinearities of the loudspeaker is to use polynomial expansion. It does have some advantages namely, computational simplicity and good approximation properties. The disadvantage is though that outside the fitted range they do not fit well. The nonlinear functions for this project were obtained using Klippel's measurement service which returned a polynomial expansion fitted for a limited range of displacement. Since these functions diverge quickly as mentioned above, it was desired to find functions which would estimate nonlinear function better outside the fitted range.[Age07]

Here follows a explanation of the kernel functions and the reasoning behind them.

Force factor $Bl(x)$

As described in Chapter 2.1 the force factor should decrease rapidly when the voice coil leaves the gap [Age07]. In other words, Bl should be at its maximum in the rest position and then decay quickly as the magnet gap is not occupied with the voice coil. One of the disadvantages of using a polynomial expansion to describe this curve is that it easily gets negative values for large displacements. One solution to overcome this and still have rapid decay outside the rest position is to use a Gaussian sum. The Gaussian sum is described below.

$$Bl(x) = \sum_{n=1}^{N} \alpha_n e^{-\frac{(x-x_n)^2}{2\sigma^2}} \tag{3.12}$$

and its derivative

$$\frac{Bl(x)}{dx} = \sum_{n=1}^{N} \alpha_n \frac{x_n - x}{\sigma^2} e^{-\frac{(x-x_n)^2}{2\sigma^2}} \tag{3.13}$$

One should be careful when choosing the Gaussian sum parameters as a bad choice can give a "bumpy" fit for small displacement. To put it differently, it is not desirable to have a curve which goes in waves for small displacement. Obliviously, it is still not guaranteed that the force factor will not have negative values but one can easily make sure of that since it happens before the function decays to zero. With the right choice of parameters, the Gaussian sum makes a good fit of what one would expect the behavior of the force factor to be outside the fitted range. A plot of the fitted force factor can be seen in Figure 3.2.

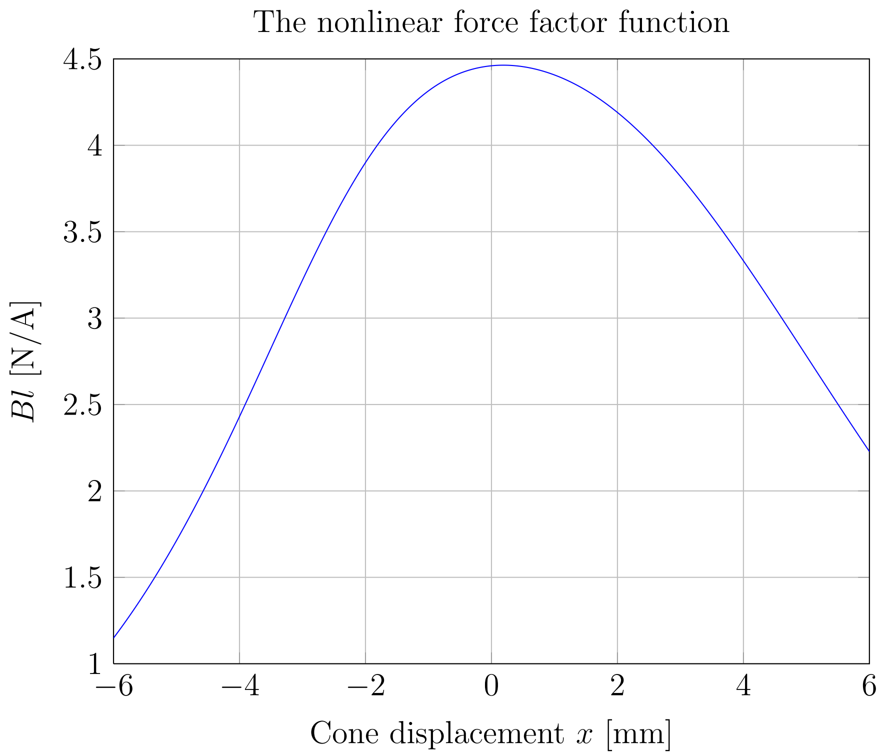

Figure 3.2: The force factor as a function of the displacement.

Voice coil inductance $L_e(x)$

As the other major nonlinearities, the voice coil inductance depends also on the displacement of the cone. This is because of when the voice coil moves forward and leaves the magnetic field, more of the coil is filled with air which reduces the inductance [Age07]. In this case it is desired to have a function which has a negative slope around rest position and should subside towards constant value for large displacement. Obviously, a polynomial fit is a bad option here, since it decays towards constant value on the x-axis. A mean to overcome this is to use a sigmoid function as described below

$$L_e(x) = \frac{L_1}{1 + e^{-a(x-x_0)}} + L_0 \tag{3.14}$$

and its first and second derivative

$$\frac{dL_e(x)}{dx} = \frac{aL_1 e^{-a(x-x_0)}}{\left(1 + e^{-a(x-x_0)}\right)^2} \tag{3.15}$$

$$\frac{d^2L_e(x)}{dx^2} = \frac{a^2 L_1 e^{-a(x-x_0)}}{\left(1 + e^{-a(x-x_0)}\right)^2} \left(\frac{2e^{-a(x-x_0)}}{1 + e^{-a(x-x_0)}} - 1 \right) \tag{3.16}$$

This gives a very good fit to the desired values at large displacement. Another advantage is that it is relatively simple and can be easily fitted. The resulting plot of the voice coil inductance can be seen in Figure 3.3.

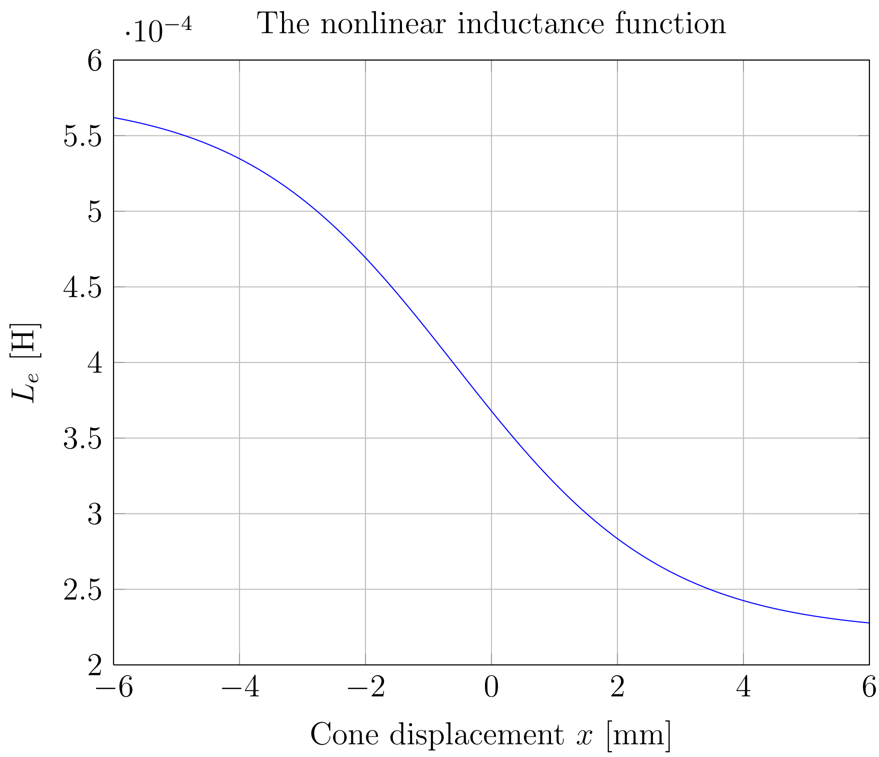

Figure 3.3: The voice coil inductance as a function of the displacement.

Suspension compliance $C_{ms}(x)$

Suspension compliance is somewhat different from the other major nonlinearities. It is accepted that the three major nonlinearities depends also on the temperature of the voice coil, but in the case of the suspension compliance this is more of a significant factor that for the other [Age07]. This can be explained by the fact that the heat of the voice coil will spread via the spider to the suspension where it will change the physical properties of the material. This temperature variations will though not be considered in this project as stated above.

Similar to the force factor it was chosen to fit the suspension compliance with a Gaussian sum. This is because it is expected that the function will have its peak value close to the rest position and then decay rather quickly with the displacement, especially in the negative direction. The Gaussian sum is stated here again for convenience.

$$C_{ms}(x) = \sum_{n=1}^{N} \alpha_n e^{-\frac{(x-x_n)^2}{2\sigma^2}} \tag{3.17}$$

Its derivative can be seen in equation 3.13. The resulting curve can be seen in Figure 3.4.

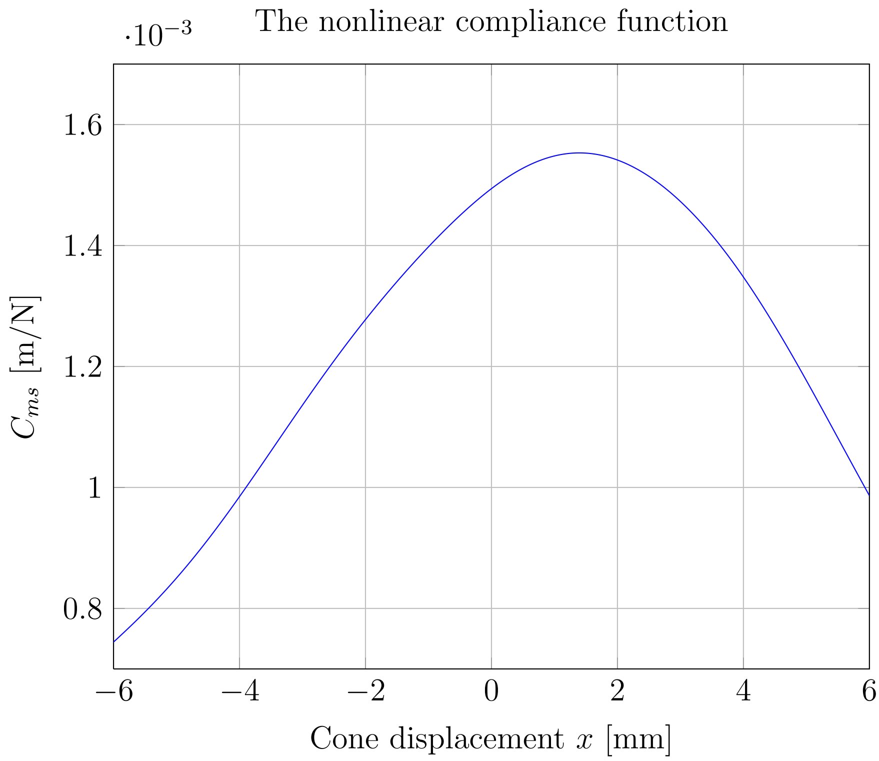

Figure 3.4: The suspension compliance as a function of the displacement.

The method used to fit those nonlinearities to their desired function was the lsqcurvefit function in Matlab. This algorithm solves the nonlinear curve-fitting problem in least-squares sense and makes for an efficient way to fit the curves to the desired values.

3.2 Controller

The function of the controller is to compensate for the nonlinear behavior of the loudspeaker and thus render the system linear. A few approaches have been tried in the past to do exactly this [Kli03]. Here it was chosen to use the theory of exact input-output linearization since it promise to give a very good results given correct feedback [SSH96].

The exact input-output linearization control law is divided into two parts, the linear dynamics (LD) and the inverse dynamics (ID). Assuming full state feedback, the control law can be represented by the block diagram in Figure 3.5. The LD and the ID block are described by equations 3.18 and 3.29 respectively.

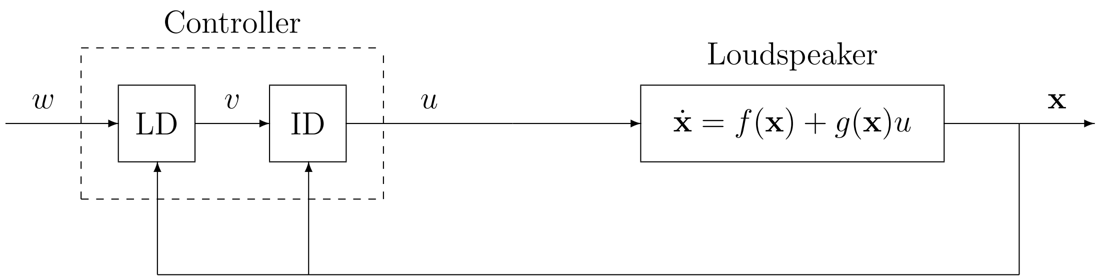

Figure 3.5: A diagram of the system assuming full state feedback.

As described in Chapter 2.2, calculating the control law for the inverse dynamics is simply done by taking the derivative of the function $\mathbf{h}(\mathbf{x})$ until it explicitly depends on the input u. Doing that, the control law for the inverse dynamics can be found and are shown below.

$$\begin{aligned} u = \Bigg\{ & Mv + \frac{x_2}{C_{ms}(x)}\left(1 - \frac{x_1}{C_{ms}(x)}\frac{dC_{ms}(x)}{dx}\right) \\ & + \frac{R_{ms}}{M}\left(\frac{-x_1}{C_{ms}(x)} - R_{ms}x_2 + \left(Bl(x) + \frac{1}{2}\frac{dL_e(x)}{dx}x_3\right)x_3 + \frac{1}{2}\frac{dL_2(x)}{dx}x_4^2\right) \\ & - x_2x_3\frac{dBl(x)}{dx} - \frac{1}{2}x_2x_3^2\frac{d^2L_e(x)}{dx^2} - \frac{1}{2}x_2x_4^2\frac{d^2L_2(x)}{dx^2} \\ & - \frac{x_4}{L_2(x)}\frac{dL_2(x)}{dx}\left(R_2(x)x_3 - \left(R_2(x) - x_2\frac{dL_2(x)}{dx}\right)x_4\right)\Bigg\}\left(\frac{L_e(x)}{Bl(x) + x_3\frac{dL_e(x)}{dx}}\right) \\ & + Bl(x)x_2 + x_2x_3\frac{dL_e(x)}{dx} + R_ex_3 + R_2x_3 - R_2x_4 \end{aligned} \tag{3.18}$$

Since it was needed to take the derivative of the function $\mathbf{h}(\mathbf{x})$ three times before u appeared explicitly, it correspond to that the system has a relative degree of three. As the nonlinear functions, $Bl(x)$, $L_e(x)$ and $C_{ms}(x)$ are known and explicitly defined, finding their derivatives is a simple task.

As the system has a relative degree of three, which is one less than the number of states, it will results in that zero dynamics will be present in the system. Since this is the case, it does make the linear dynamics control law a little less straightforward than otherwise. To find the LD control law, the state parameters need to to be transformed where one of the transformed states is the cone acceleration a and where it is possible to get an understanding of the poles of the transformed system. This is done because it is desirable to be able to choose the performance of the linear system.

Choosing the transformation is straightforward for the first three states but the forth needs a little more consideration. First the associative state transformation is defined as

$$\mathbf{z} = \begin{bmatrix} x_1 & x_2 & \dot{x}_2 & \Psi \end{bmatrix}^T \tag{3.19}$$

where Ψ can be chosen freely as long as equation 3.20 holds [SL91].

$$\mathbf{L_g}\Psi = 0 \tag{3.20}$$

This yields that

$$\mathbf{L_g}\Psi = \frac{1}{L_e(x)}\frac{\partial \Psi}{\partial x_3} = 0 \tag{3.21}$$

Ψ can therefore freely be chosen as x_4 for simplicity. The transformation will therefore be

$$\mathbf{z} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ i_2 \end{bmatrix} \tag{3.22}$$

Carrying out the calculation, the new state vector $\mathbf{z}$ dependence on the former state vector $\mathbf{x}$ can be seen below.

$$\begin{aligned}
z_1 &= x_1 \\
z_2 &= x_2 \\
z_3 &= \frac{1}{M}\left(\frac{-x_1}{C_{ms}(x)} - R_{ms}x_2 + Bl(x)x_3 + \frac{1}{2}\frac{dL_e(x)}{dx}x_3^2 + \frac{1}{2}\frac{dL_2(x)}{dx}x_4^2\right) \\
z_4 &= x_4
\end{aligned} \tag{3.23}$$

The transformed state space can now be formulated and a new input w added. This is shown in an convenient manner in equation 3.24. The parameters $p_1, \ldots, p_4$ are approximate of how the $\dot{z}_4$ depend on $\mathbf{z}$. The function $\dot{z}_4$ is though nonlinear and not additive and will be stated for now as below for convenience and for simplicity.

More trivially, the parameters $k_1, \ldots, k_3$ are some of the coefficients of the characteristic polynomial of the desired linear system and can be chosen arbitrarily. How those parameters are chosen will define the linear performance of the system.

$$\dot{\mathbf{z}} = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -k_1 & -k_2 & -k_3 & -k_4 \\ \hline p_1 & p_2 & p_3 & p_4 \end{array}\right]\mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} w \tag{3.24}$$

To understand the choices made for those poles, equation 3.25 is shown for explanation.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & 0 \\ \mathbf{A}_{21} & \mathbf{A}_{21} \end{bmatrix} \tag{3.25}$$

Since $\mathbf{A}_{11}$ is in phase variable canonical form in equation 3.24 the coefficients $k_1, \ldots, k_3$ will define the poles for it. To get equation 3.24 in the same form as equation 3.25, k_4 is chosen as zero. This is necessary to have an understanding of the poles which define the system. The last pole is the p_4 since the poles of $\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ define the poles of $\mathbf{A}$ in equation 3.25.

To find the coefficients $k_1, \ldots, k_3$ for the linear behavior, the loudspeaker model needs to be linearized. This is a simple task where the nonlinear functions are replaced with their values at rest position, $(x = 0)$. The resulting $\mathbf{A}$ matrix for the linear state space form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$ can be seen in equation 3.26.

$$
\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{MC_{ms}} & \frac{-R_{ms}}{M} & \frac{Bl}{M} & 0 \\ 0 & \frac{-Bl}{L_e} & \frac{-R_e - R_2}{L_e} & \frac{R_2}{L_e} \\ 0 & 0 & \frac{R_2}{L_2} & \frac{-R_2}{L_2} \end{bmatrix} \tag{3.26}
$$

Finally, the characteristic equation of the linearized system is found

$$
det(\lambda\mathbf{I} - \mathbf{A}) = s^4 + k_4 s^3 + k_3 s^2 + k_2 s^1 + k_1 \tag{3.27}
$$

where the coefficients of the characteristic polynomial are shown in below.

$$
\begin{aligned}
k_1 &= -\frac{R_2}{L_2}\left(\frac{R_2}{L_e^3}(R_2+R_e)^2 - \frac{Bl^2 R_2}{ML_e^2}\right) + \frac{R_2^2}{L_2 L_e}\left(\frac{1}{MC_{ms}} + \frac{Bl^2 + R_{ms}(R_2+R_e)}{ML_e}\right) \\
&\quad - \frac{R_2^2}{L_2 L_e^2}(R_2+R_e)\left(\frac{R_2+R_e}{L_e} + \frac{R_{ms}}{M}\right) \\
&\quad - \frac{R_2}{C_{ms}L_2 L_e M}(R_2+R_e) \\
k_2 &= \frac{R_2}{L_2}\left(\frac{1}{MC_{ms}} + \frac{Bl^2 + R_{ms}(R_2+R_e)}{ML_e}\right) - \frac{R_2^2}{L_2 L_e}\left(\frac{R_2+R_e}{L_e} + \frac{R_{ms}}{M}\right) \\
&\quad + \frac{R_2+R_e}{MC_{ms}L_e} + \frac{R_2^2}{L_2 L_e^2}(R_2+R_e) \\
k_3 &= \frac{1}{MC_{ms}} + \frac{R_2}{L_2}\left(\frac{R_2+R_e}{L_e} + \frac{R_{ms}}{M}\right) - \frac{R_2^2}{L_2 L_e} + \frac{Bl^2 + R_{ms}(R_2+R_e)}{ML_e} \\
k_4 &= 0
\end{aligned} \tag{3.28}
$$

The resulting control law for the linear dynamics can be seen in equation 3.29, where $\mathbf{z}$ dependency on $\mathbf{x}$ can be seen in equation 3.23 and w is the new input.

$$
v = -k_1 z_1 - k_2 z_2 - k_3 z_3 - k_4 z_4 + w \tag{3.29}
$$

Obviously, this control law needs to be implemented before any amplification to the loudspeaker. The desired amplification of the signal needs therefore to be multiplied with the input w. Calculating the output u with the control law described above, results then in the actual terminal voltage of the loudspeaker. The hardware needs therefore to be calibrated so that the output u is amplified to its exact value.

Another consideration concerning the hardware is that the linear dynamics can be too fast for the D/A converter. This does not have a harmful influence to the system but rather it is unnecessary to have faster dynamics than the hardware can handle. Since the poles can be chosen to represent the dynamics of the desired system it is easy to make the system slower than previously stated. This is done simply be dividing the poles with a common constant.

Now when the control law has been calculated and it is known that zero dynamics is present in the system, the question left unanswered is, what is the zero dynamics of the system. Since the zero dynamics cannot be seen in the output of the system, it is necessary to make sure that it does not have harmful influence to the system.

The internal dynamics related to the input-output linearization is simply the derivative of the choice of Ψ. This is shown below where $\dot{\Psi}$ can be described by

$$\dot{\Psi} = f(z_1, z_2, z_3, \Psi) \iff \dot{z}_4 = f(\mathbf{z}) \tag{3.30}$$

In other words, this corresponds to the last row in equation 3.24 which was defined linearly before for simplicity . The zero dynamics is then found simply be putting the states $z_1, \ldots, z_3$ to zero as described below. [SL91]

$$\dot{z}_4|_{(0,0,0,z_4)} = f(0, 0, 0, z_4) \tag{3.31}$$

Putting the input w to zero ensures that the output always equal zero. This means that the states $z_1, \ldots, z_3$ are also equal to zero. Carrying out that calculation gives that the zero dynamics are

$$\begin{aligned}\dot{z}_4|_{(0,0,0,z_4)} = {} & \frac{R_2(0)}{L_2(0)} \left(\frac{-Bl(0)}{\frac{dL_e(0)}{dx}} \overset{(+)}{-} \sqrt{\left(\frac{Bl(0)}{\frac{dL_e(0)}{dx}} \right)^2 - \frac{1}{\frac{dL_e(0)}{dx}} \frac{dL_2(0)}{dx} z_4^2} \right) \\ & - \frac{1}{L_2(0)} \left(R_2(0) + \frac{dL_2(0)}{dx} \right) z_4 \end{aligned} \tag{3.32}$$

which gives two solutions to the problem. Only one of them does though represent the physical system, which was found to be the one with the minus sign.

A few approximations were though made to this system as described in Chapter 3.1.2 and since $L_2(x)$ was approximated to a constant, its derivative equals to zero. This simplifies the analysis of the zero dynamics significantly, which can be seen with the approximation below.

$$\dot{z}_4|_{(0,0,0,z_4)} = -\frac{R_2}{L_2} z_4 \tag{3.33}$$

Since the constants R_2 and L_2 are positive the zero dynamics are clearly stable.

3.3 State estimation

Since full state feedback is required for the controller to work, a state estimation has to be made. This is due to the restriction that all of the states can not be measured under a normal working scenario. Two state estimation methods where implemented which both have their advantages and disadvantages. Those two are the feed-forward state estimation and the observer based state estimation.

3.3.1 Feed-forward state estimation

The feed-forward state estimation is much simpler than the other. It is not based on any measurement, but simply, the loudspeaker model is used as reference. This means that the only thing that is needed to do, is to take the input signal u and the state vector $\hat{\mathbf{x}}$ and calculate the new state vector $\hat{\mathbf{x}}$ as can be seen in equation 3.10. This is illustrated in Figure 3.6.

The means to calculate the derivative was chosen to be the Runge-Kutta algorithm of order 4. There are many means to numerically integrate the differential equation but this was chosen since Runge-Kutta is assumed a good comprise between simplicity and robustness [Bra92]. The performance varies greatly with the chosen value of the step length h, so care must be taken to ensure stability of the algorithm.

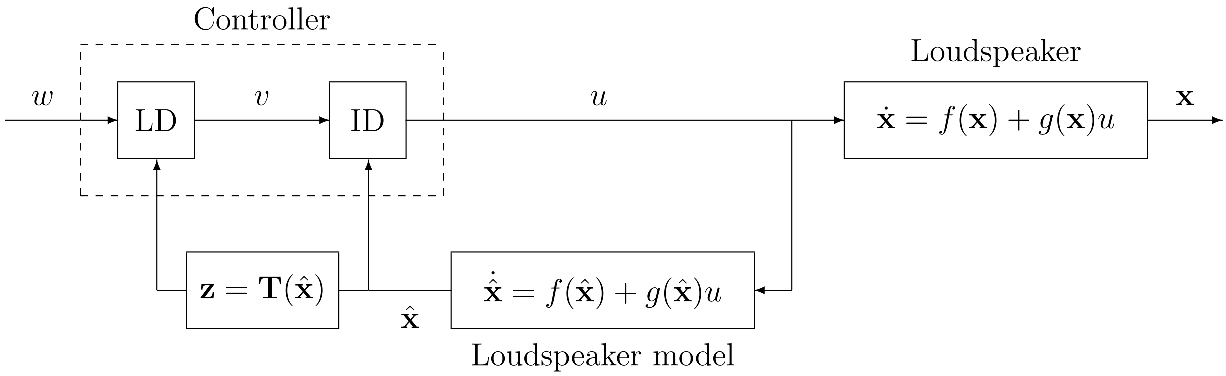

Figure 3.6: A diagram of the feed-forward system.

The advantages of using this method is that no additional setup is needed since this is just applied to the input signal of the loudspeaker. The disadvantage is that it has no means of correcting it self when it is faced with model errors. In other words, if the loudspeaker model varies from the reality, the feed-forward state estimation will have no means of correcting itself with respect to that difference.

3.3.2 Observer based state estimation

Due to the process noise vulnerability of the feed-forward state estimation, an observer based one was also considered. There are many choices to consider here but the unscented Kalman filter (UKF) was chosen for its alleged performance to estimate nonlinear systems.

Other observers where also considered, including its relative, the extended Kalman filter. The unscented Kalman filter is though considered better if faced with strong nonlinearities and has little additional complexity since it does not have to compute the Jacobian nor the Hessian of the model. [JS10]

To illustrate the observer role in the whole system, the figure of the observer based system is shown below again for convenience. As can been seen the observer uses the input u and the measured current x_3 to estimate the states $\hat{\mathbf{x}}$.

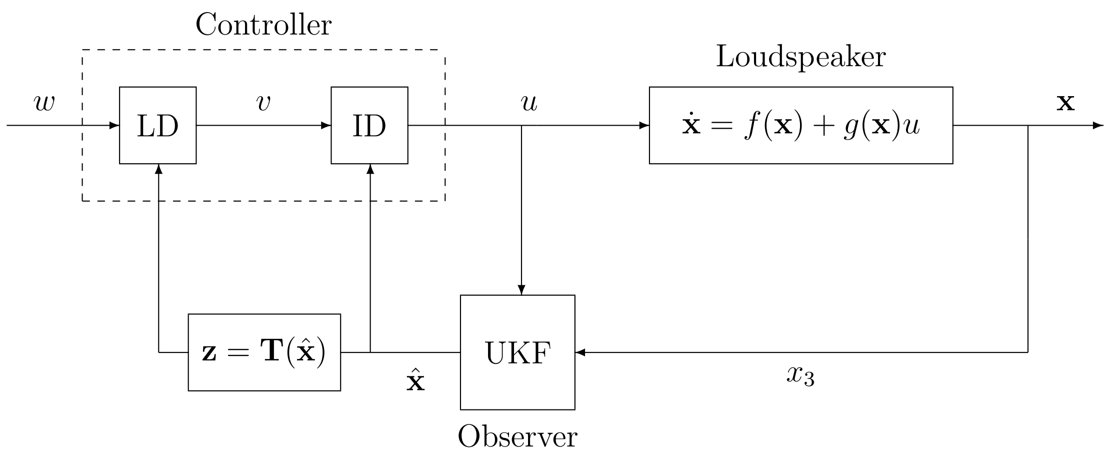

Figure 3.7: A diagram of the observer based system.

The unscented Kalman filter was implemented as described in Chapter 2.3. This is the non-augmented version of the UKF which assumes that the noise is additive. It is simpler than the augmented one, since it has a smaller number of sigma points of which it has to evaluate. This was chosen as a basis to start with, but no comparison was made with those two in this case.

Like the feed-forward system discussed earlier, the integration of the differential equation as described in equation 3.10 needs to be calculated since the UKF makes use of the loudspeaker model as well. As before, it was chosen to use the Runge-Kutta algorithm of order 4 to carry out the calculation. This means that the stability of the observer is also dependent of the chosen step length h.

Instead of the actual states, it is the sigma points which are propagated through the loudspeaker model. As described in Chapter 2.3, those sigma points are used to estimate the means and covariance of the states. How the UKF algorithm works in detail can be seen in Chapter 2.3.

There are a number of options to tune the unscented Kalman filter based on the environment in which it has to work in. To start with, it has the positive parameters α, β and κ. The parameters α and κ control the spread of the sigma points where as β can be used to incorporate prior information on the distribution of $\mathbf{x}$. In other words, those parameters control the weight matrices $\mathbf{W}_c$ and $\mathbf{W}_m$ which in turn decide how well the UKF captures higher order movements of the loudspeaker model. [KFI08]

Much like its simpler relative, the Kalman filter, it has dedicated tuning parameters for the covariance of the process noise $\mathbf{Q}$ and covariance of the measurement noise $\mathbf{R}$. Since the measurement noise can be assumed small when only measuring the current x_3, the covariance of the measurement noise $\mathbf{R}$ can also be chosen small. More significantly, the covariance of the process noise $\mathbf{Q}$ has to be chosen as such to represent the difference between the loudspeaker model and the real loudspeaker. Tuning only this parameter can make a significant difference on the performance of the observer.

Like the Kalman filter, the initial guess for the mean $\mathbf{m}_0$ and the initial covariance matrix $\mathbf{P}_0$ has to be chosen. In this case, this is considered a simple task since under no stimulus the state values should be equal to zero. Similarly, the covariance matrix $\mathbf{P}_0$ can be chosen small because of the little uncertainty.

4 Results

The loudspeaker model, controller and state estimation were implemented in Matlab/Simulink to validate the performance. Additionally, measurements were done on a real loudspeaker for model and controller verification.

This chapter will first compare how well the loudspeaker model agrees with the loudspeaker and then give an example of how the controller performs. Additionally, this chapter will describe in detail how the system is affected by parameter variations for the two different state estimation methods. Finally, the experimental results are given for the feed-forward controller.

Human hearing perceive sound by listing for pressure changes in the channel medium. As the pressure is produced by the acceleration of the cone, it will be the output vector of which the results will be based on [Ben93].

4.1 Model verification

Since the moving coil loudspeaker is a nonlinear system it is quite difficult to analyze which of the parameters are the ones responsible for the difference between the loudspeaker model and the loudspeaker, assuming the model equations are accurate.

Because of this, it was decided to use the harmonics of the acceleration of the cone as a measure of how well the model fitted the loudspeaker. For this purpose the loudspeaker and the model was fed with a sinusoidal wave with different amplitude at its terminal to measure the cone response.

In Simulink the model was also simulated under the same experimental conditions. The loudspeaker model was simply fed with a sinusoidal wave with different amplitudes to simulate the terminal voltage and then the acceleration was saved before it was integrated to the cone velocity, x_2. As described earlier in Chapter 3.1.2 the parameters for the loudspeaker was given by Klippel's measurement service and the nonlinear parameter was then fitted to the kernel functions described in Chapter 3.1.2.

To keep the comparison as accurate as possible the same loudspeaker was used for the experimental case. To acquire the cone response for the experimental measurement required a little more elaborate equipment. To get the corresponding data from the loudspeaker a sinusoidal wave was fed from a signal generator via an amplifier to the loudspeaker. An oscilloscope was used to calibrate the terminal voltage to the desired amplitude. To measure the cone response, a laser doppler vibrometer (LDV) was used to measure the velocity of the cone. It automatically provides the derivative of the data to yield the acceleration. To acquire the more interesting part of the measurement a Hanning window was used as well as a low-pass filter [win10].

The measurements and simulation can be seen in Figure 4.1. The plots are divided into multiples of the fundamental frequency dependent on the frequency for different terminal voltage. The the first 5 multiples of the fundamental were selected to serve as a results since it must obviously be limited to a finite number and the higher order harmonics were barely noticeable.

In Figure 1.1 an illustrated explanation can be seen of what harmonic distortion is. A more detailed description is though that the n:th order harmonic distortion of the fundamental frequency f_1 is defined as

$$\mathrm{HD_n} = \frac{|P_n|}{P_1} \cdot 100\% \qquad (4.1)$$

where P_n is the rms sound pressure at the n:th harmonic and P_1 is the rms-value of the

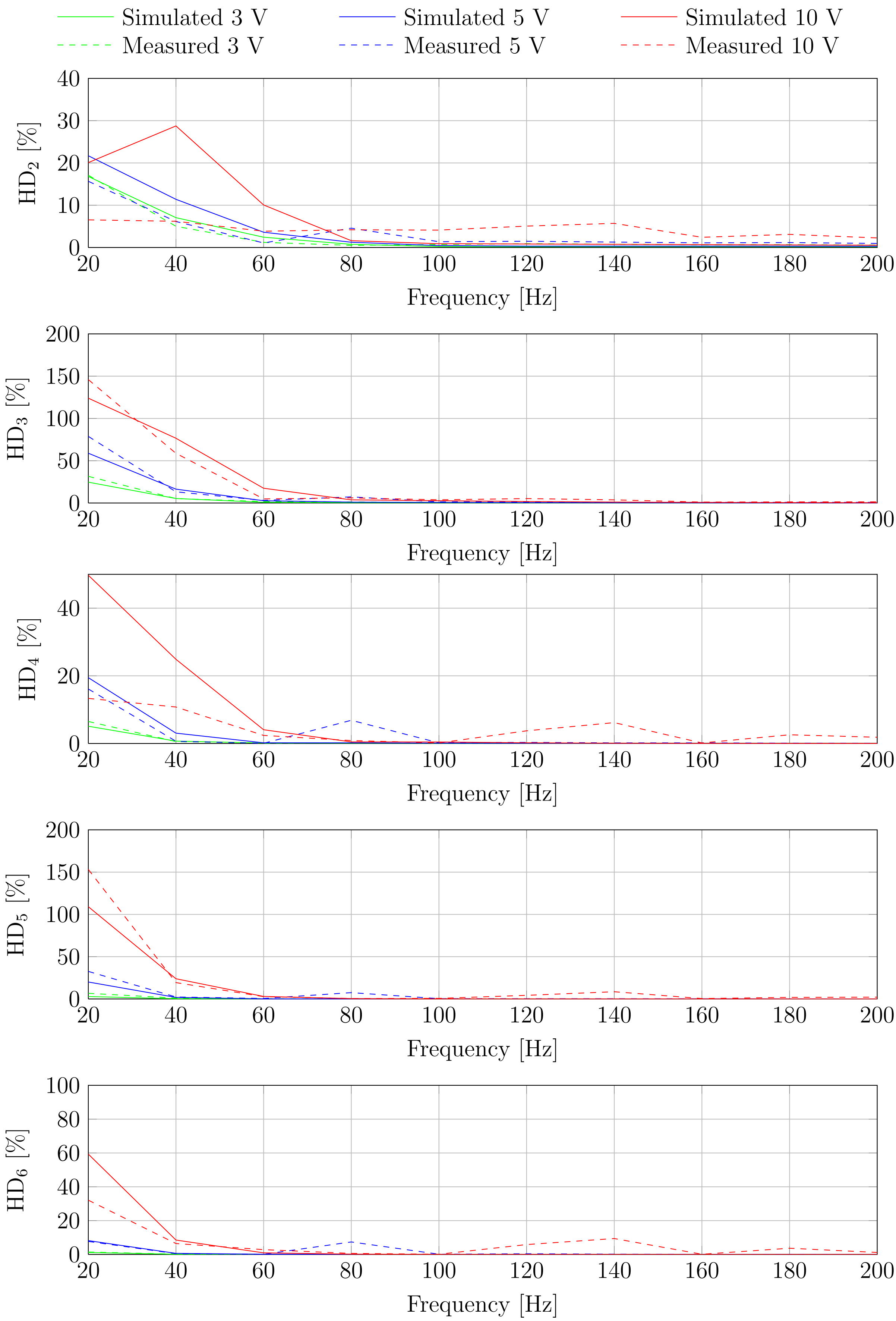

Figure 4.1: The harmonics of the loudspeaker model and the loudspeaker when fed with sinusoidal wave with different amplitudes.

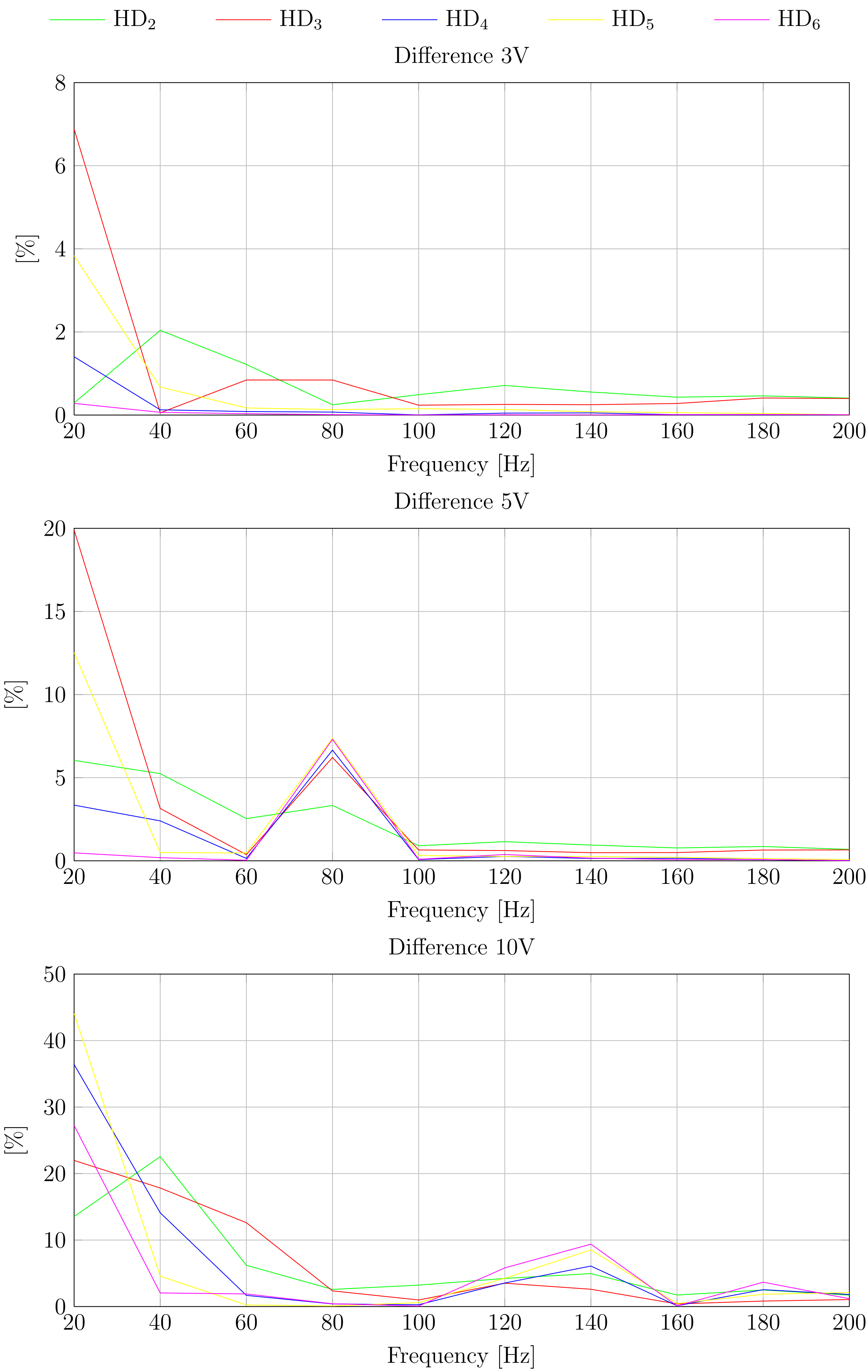

Figure 4.2: The harmonic difference between the loudspeaker model and the loudspeaker when fed with sinusoidal wave with different amplitudes.

fundamental frequency. A more common method for measuring HD and THD in audio application is to use the (T)HD-N definition as given by the IEC standard 60268-5 [iec03].

Since this work focuses solely on the harmonics and its sources this standard was considered not suitable.

In Figure 4.1 the harmonic distortion is shown for the measured and simulated loudspeaker and in Figure 4.2 the difference between them are shown. As can be seen, the difference increases considerably when the terminal voltage is increased. This is due to the fact that the cone excursion increases considerably as well which increases the possibility of the nonlinear function error. In Figure 4.1 it can also be seen how much the harmonics increases below the resonance frequency, which is 63,1 Hz. Over the resonance frequency the harmonics are considerable less which is consistent for both the simulated and measured loudspeaker. It should also be noted what looks likes a faulty measurement for 80 Hz 5V which is shown clearly in Figure 4.2.

In Figure 4.1 it can also be seen how much larger the odd order harmonics are than the even order. This can be seen clearly for the 2:nd and 3:rd order harmonics. This indicates that the symmetrical nonlinearities are much stronger than the asymmetrical for the loudspeaker [Kli06]. This is consistent for both the loudspeaker model and the loudspeaker.

All in all, the difference between the loudspeaker model and the loudspeaker should be considered significant even though they are able to show similar behavior. This difference increases with larger terminal voltage or in other words with larger cone excursion.

4.2 Parameter sensitivity

Similarly as above, the total harmonic distortion (THD) is the sum of all harmonic components divided with the fundamental. Since a linear system will produce no harmonics when fed with a sinusoidal wave, this definition will serve well to explain the overall performance of the nonlinear system. The equation for the THD can be seen below.

$$\mathrm{THD} = \frac{\sqrt{\sum_{i=2}^{N} |P_i|^2}}{P_1} \cdot 100\% \tag{4.2}$$

To find out how sensitive the system is to parameter variation a number of simulations were done were the system was fed with sinusoidal wave with different amplitude and frequency. Those simulations were done using the system with the LD controller poles divided with a common factor equal to 1000. This was done since the system otherwise takes a very long time to simulate but will also serve to simulate a limitation on the hardware.

The structure for this simulation were that every loudspeaker parameter was varied between -15% and 15% but kept constant in the controller. The maximum and minimum THD for those simulations can be seen in Figure 4.3. The reason for varying the parameters between -15% and 15% is that some parameters showed to be much more sensitive when overestimated than underestimated and likewise other parameters showed to be more sensitive the other way around.

4.2.1 Feed-forward state estimation

In Figure 4.3 the THD for the loudspeaker parameter variation can be seen as a function of the terminal voltage for a number of frequencies. In the left column the nonlinear parameters can be seen and in the right the loudspeaker constants. As can be seen for the nonlinear functions, it's the force factor Bl which has the most effect of causing THD for all frequencies. The suspension compliance C_{ms} has also large effect but it decreases considerably with increasing frequency. The voice coil inductance L_e causes on the other hand quite small effect but can be seen increasing for higher frequencies.

For the loudspeaker constants it can be seen that it is the cone mass M which has the largest effect of causing THD. The suspension mechanical resistance R_{ms} does also cause large effect for very low frequencies but does decrease rapidly for increasing frequencies. The voice coil resistance R_e does on the other hand have small effect for lower frequencies but increase rapidly for higher frequencies.

Since the poles of the LD controller was divided with a common factor it causes the controller to behave slower than optimal. This causes the controller to never be able to fully get rid of all harmonics, even though it does reduce them significantly. This can be seen as the minimum value plots in Figure 4.3. Another aspect to the minimum values is that many of them are not found for zero variations. In other words, when there is no variation between the controller parameters and the loudspeaker. This can be explained and was verified to be because some of the parameter changes were causing the loudspeaker to have more linear behavior.

Another thing that is common for all of the loudspeaker parameters is that the THD is considerably larger for lower frequencies than higher.

4.2.2 Observer based state estimation

Results similar to the feed-forward case can be seen in Figure 4.4 for the observer based system. This was expected since it is still the same parameters which has the largest effect on the system.

The simulations for the observer based system were done using the same tuning parameters. Acceptable settings were found for the UKF by a series of simulations. Those common settings were used for all parameter variation simulation. Those settings showed to be acceptable for most of the parameters but on the other hand, one can notice considerable lack of performance for the voice coil resistance R_e in comparison to the feed-forward case. The reason for this performance degradation was found to be because of the covariance matrix $\mathbf{Q}$, as the optimal values for the covariance matrix was not the same for R_e as it was for the rest of the parameters. One could though find common settings which gave excellent performance for all parameters, although not optimal for all. It will though not be shown here.

If Figures 4.3 and 4.4 are compared, one can see that the observer based system performs better for low frequencies but the difference tends to decrease for higher frequencies.

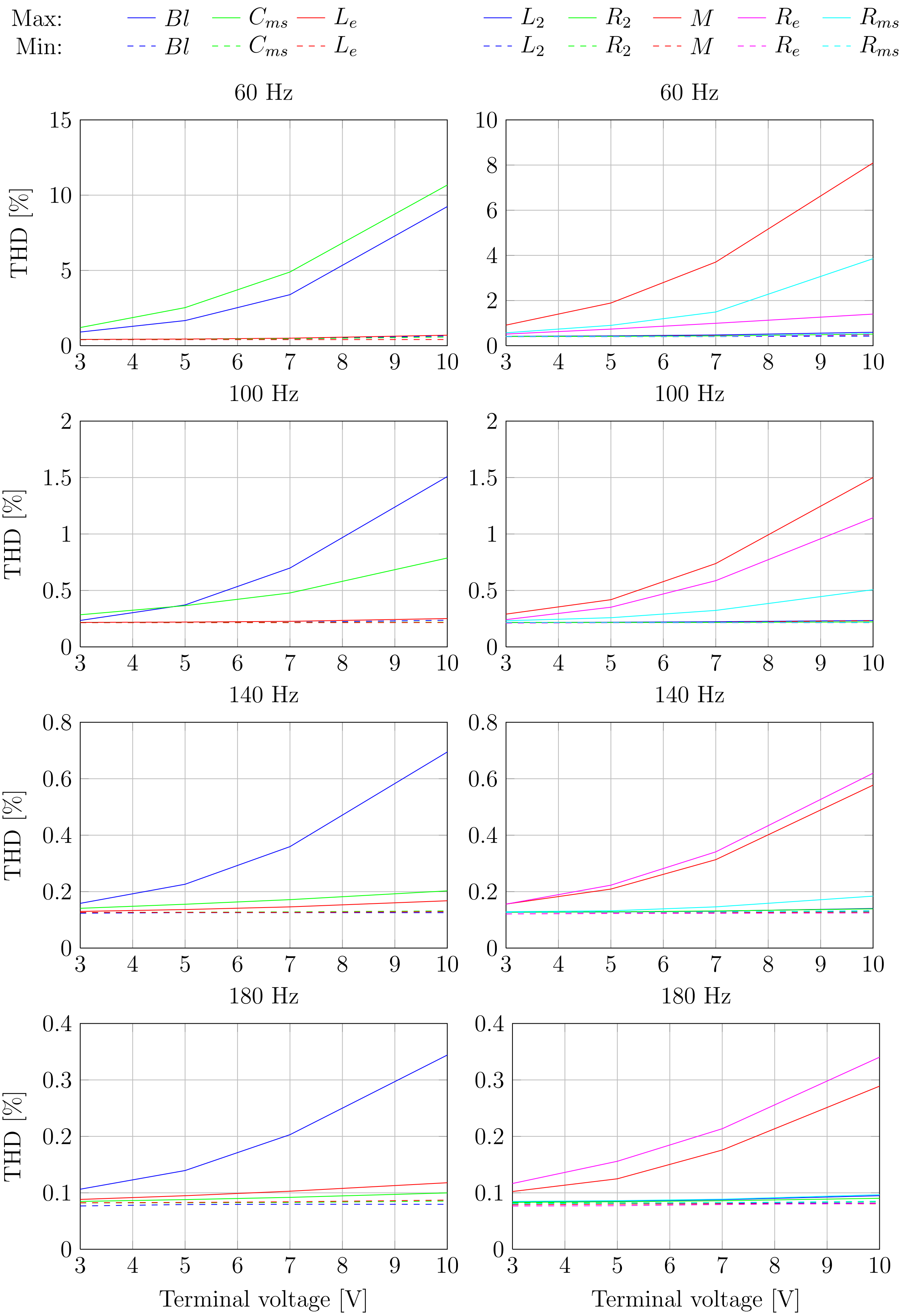

Figure 4.3: THD simulated with parameter variation for the feed-forward system.

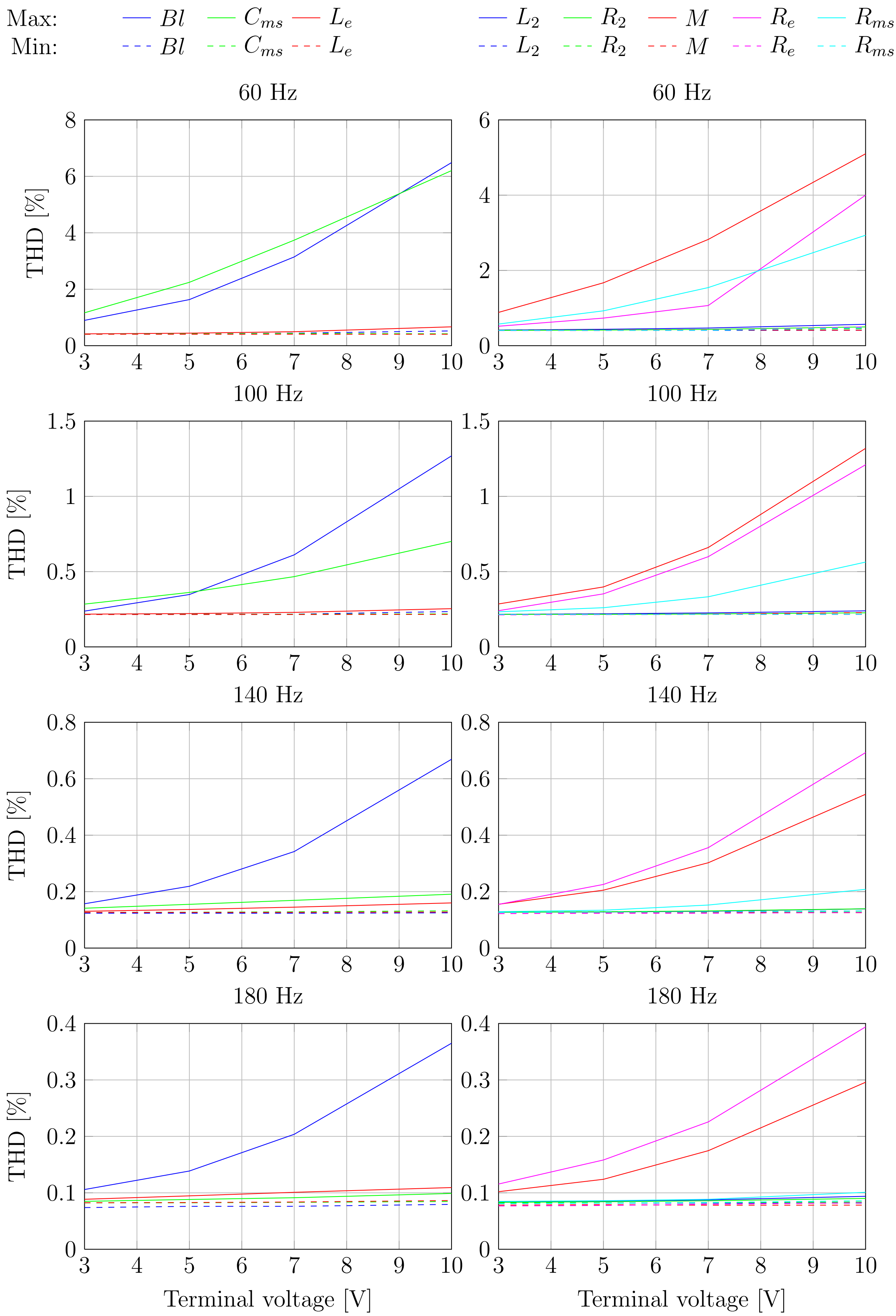

Figure 4.4: THD simulated with parameter variation for the observer based system.

4.3 Controller performance

To get an understanding of how well the observer based and feed-forward system performs, a series of simulations were done. As before the stimulus is a sinusoidal wave with a different amplitude and frequency and the poles of the LD controller is divided with a common factor equal to 1000. First the observer and feed-forward system will be evaluated without process noise. That is, how well the system works if the parameters in the controller match with those in the loudspeaker. Secondly, processes noise were added and the observer and feed-forward system performance compared to the loudspeaker without controller.

4.3.1 Not affected by process noise

In Figure 4.5, one can see how the controller perform in comparison with the loudspeaker without controller. As can be seen the observer based and the feed-forward estimator perform identical for all simulation. This was expected since without process noise present, both the state estimators should perform identically. Since the poles of the LD controller were divided with a common factor, the controller is made slower than optimal. This can be seen on the plots as the controller is not able to suppress the THD to zero. It does though come close to zero which is considered good performance.

Looking at the plots, one can see the THD difference between the controlled and uncontrolled system. As can be seen the difference is substantial for the lower frequencies but decreases with higher frequencies. Since the loudspeaker performs better for higher frequencies this is quite expected. More interestingly, one can see how well the controlled system is able to perform even for large cone excursion.

In Figure 4.6 it can be seen how the controller affects the rms value of the power at the loudspeaker terminal. This is interesting since one need to make sure that the dedicated amplifier is able to dispatch the power which the controller specifies. More over, the loudspeaker physical limits can not be exceeded either. In other words, one need to make sure that the dispatched power does not harm the loudspeaker.

As can be seen in the figure, the least power is needed when the loudspeaker is operating near its resonance frequency, which is 63,1 Hz. Over its resonance frequency, no more power seems to be needed than without controller which is understandable since with increasing frequency, the HD does decrease substantially. More interesting, is the lower frequencies under its resonance frequency. There, as can be seen in Figure 4.1 the HD does increase with lower frequencies which in turn requires more control signal activity. This can be seen as increasing power for the controller in Figure 4.6.

Since the loudspeaker model is quite asymmetrical, a substantial power is needed to achieve the same cone excursion in negative direction compared to the positive one. This can be seen as quite high power requirements for 40 Hz and 10 V.

4.3.2 Affected by process noise

In Figure 4.7 it can be seen how the process noise affects the observer based and the feed-forward system. To simulate the process noise all of the loudspeakers parameters were varied manually. This was done to try to simulate real variations in the loudspeaker. Moreover, the parameter constants were varied so that the ones that is estimated to be hard to measure were varied more than the others. The nonlinear functions were changed so that they would behave similar to the ones in the controller but still have their own properties. This is different from what was done in Chapter 4.2 where the whole function was offset with the same constants. To do this, new points were simply fitted to the same kernel functions.

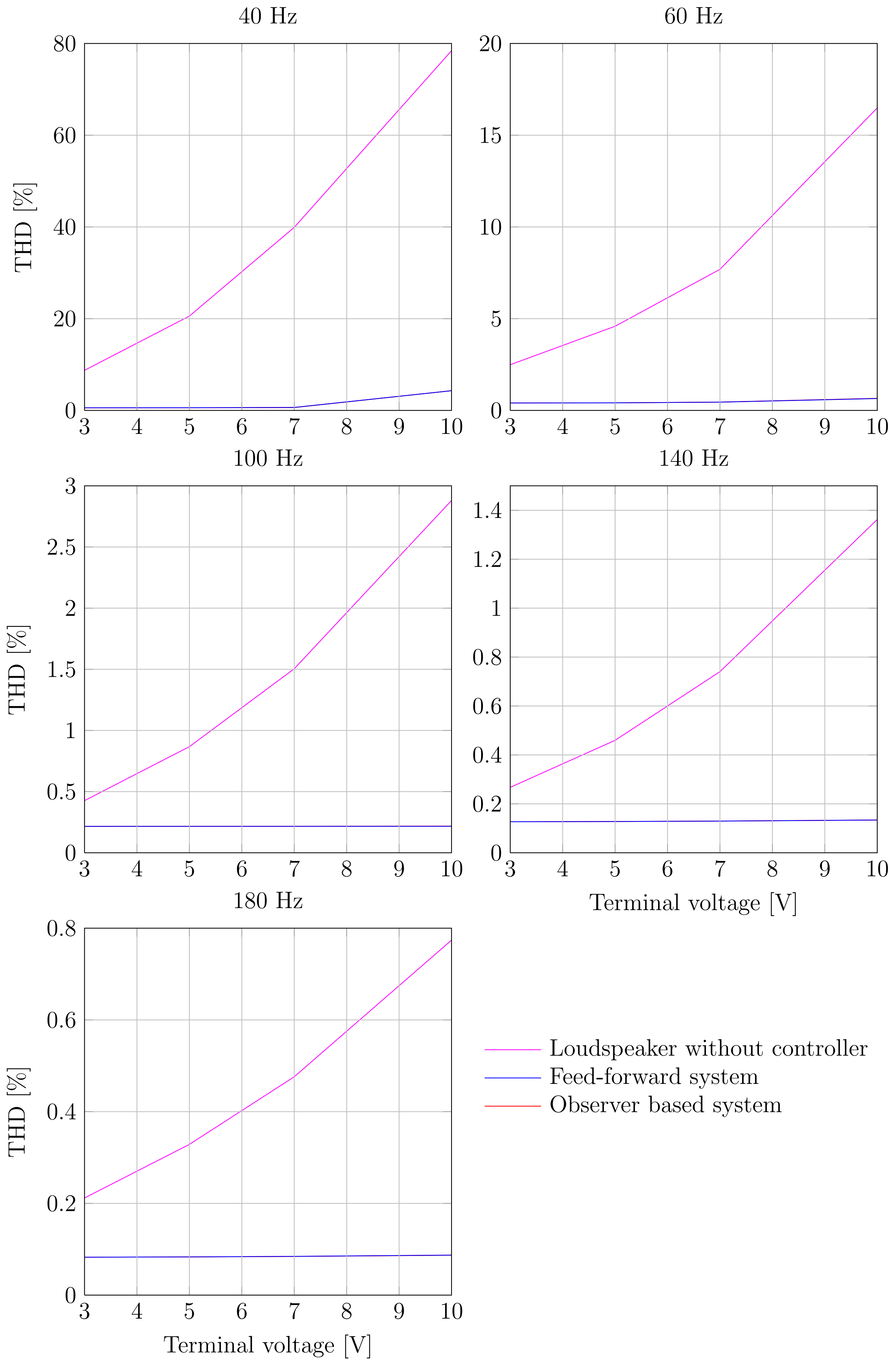

Figure 4.5: THD for the system without process noise.

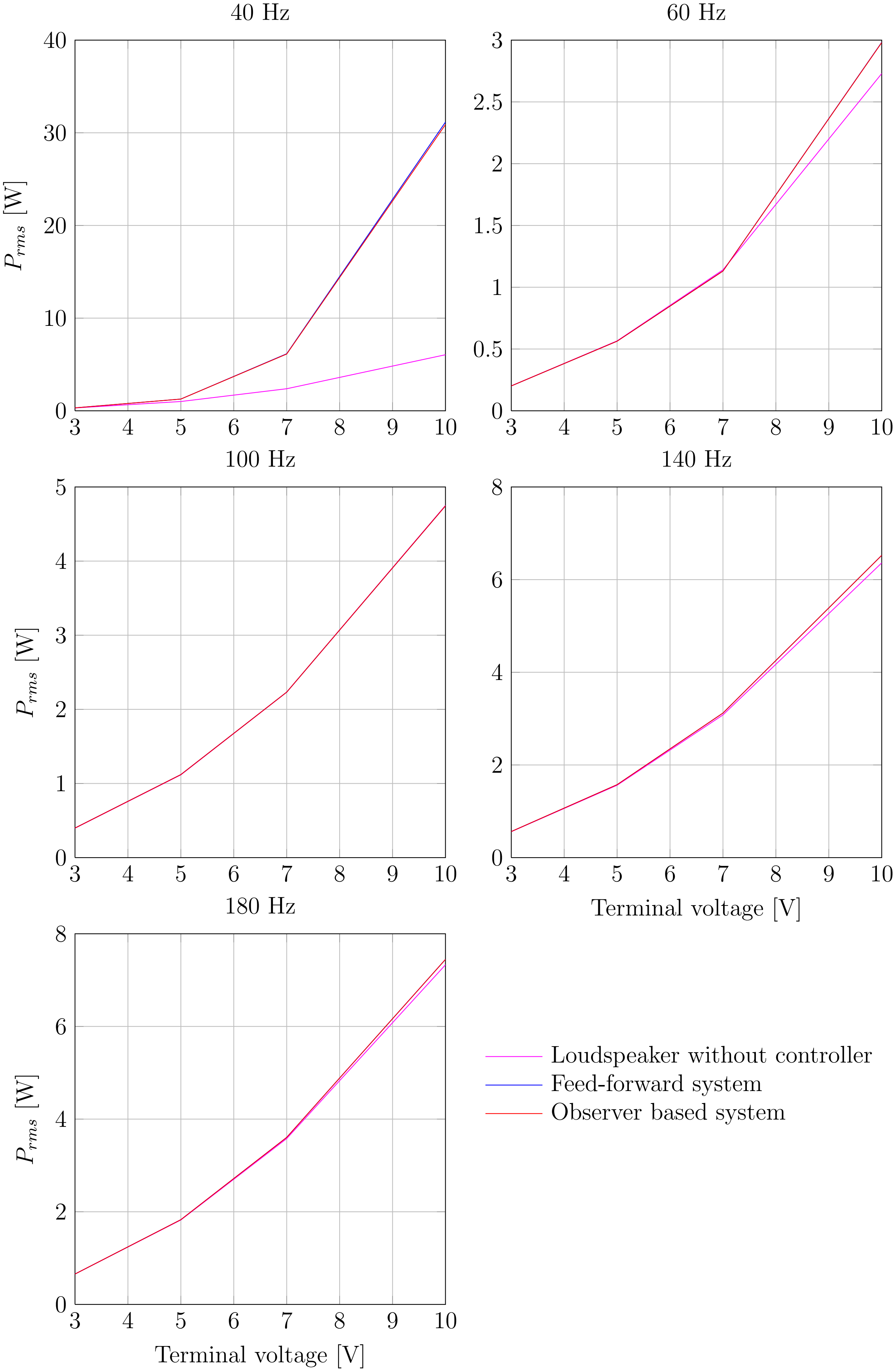

Figure 4.6: P_{rms} at the loudspeaker terminal when not affected by process noise.

Those simulations were done for a number of systems as can be seen in Figure 4.7. The loudspeaker without controller plots show how the THD values of the driver without a controller are affected by increasing terminal voltage and frequency. Similarly, the feed-forward system plots show how the driver with the feed-forward controller performs for the same terminal voltage and frequency.

Three different simulations were done for the observer based system. The difference between them is how the values for the covariance of the process noise **Q** were chosen. The **Q** 60 Hz one, were chosen so that the observer based system would be optimized for 60 Hz. In other words, the **Q** matrix was chosen so that the THD would be kept at minimum for 60 Hz and then the simulation for other frequencies were done with the same **Q** values. Identical to that, the same were done for 180 Hz which are shown in the plots as **Q** 180 Hz. The **Q** standard one, is the same **Q** matrix which was used in the parameter sensitivity simulation.

As can be seen in Figure 4.7, the standard **Q** observer and the feed-forward system have almost identical results. For low frequencies they are both able to reduce the THD considerably of which the difference increases with the terminal voltage. As one could have guessed, the observer based system with **Q** values optimized for 60 Hz performs excellent for low frequencies but its performance does worsen for higher frequencies. Similarly, the observer based system with **Q** values optimized for 180 Hz performs best of the controllers for higher frequencies but worst for the lowest ones. As one can see, the values chosen for the **Q** matrix does have significant effects on the results.

Another interesting aspect is that, here, the process noise can be said to be extensive since none of the controllers performs well at higher frequencies. In fact, the controllers performs destructively at higher frequencies for this certain type of process noise. This was though estimated to be the case and does in effect prove that a considerable accurate loudspeaker model is necessary for the controller to have positive effect for every situation.

4.4 Controller experimental results

A few experiments was performed to measure the feed-forward controller performance. The reason for that only the feed-forward system was measured was that the experimental setup is much simpler than for the observer based system. This is because the observer based system requires a feedback loop and a dedicated hardware which can handle the real-time performance demand.

Since the feed-forward system requires no feedback it was possible to evaluate its performance. The experimental setup which was used was identical to the one described in Chapter 4.1 except for the sound source which was an external sound card connected to a laptop. As before the amplification needed to be calibrated for the terminal voltage to be correct.

To carry out the experimentation, the feed-forward system was then simulated in Simulink for the estimated loudspeaker parameters and the control signal u recorded. The control signal was then played from Matlab through the external sound card.

The results from those experimentations were quite inconclusive. For a certain terminal voltage and frequency it was possible to suppress the third order harmonic somewhat but on the other hand a noticeable gain could be seen for the second order harmonics.

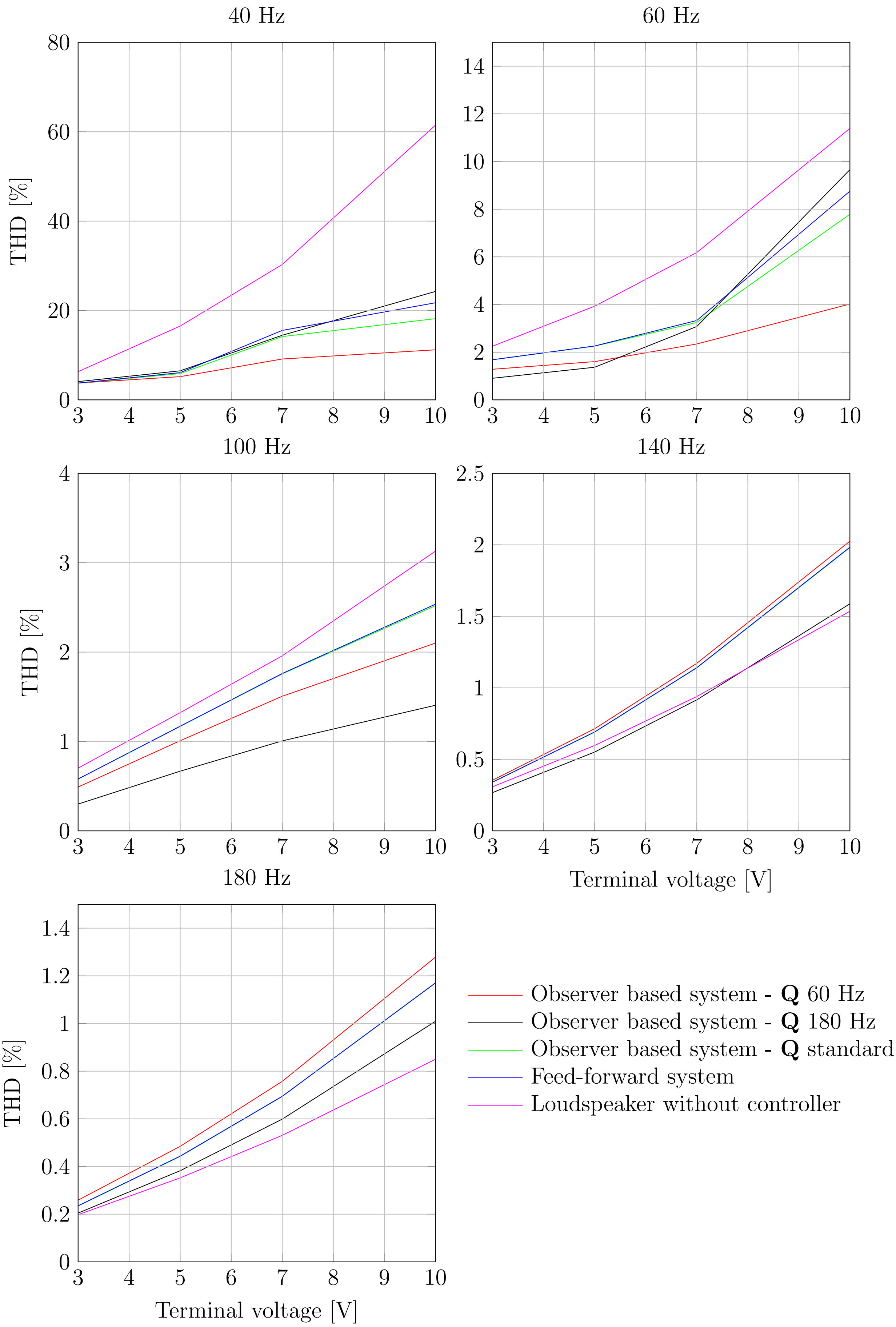

Figure 4.7: THD for the system when affected by process noise.

5 Discussion

As were found in Chapter 4.2 the loudspeaker parameters do affect the linearity differently. The results for the nonlinear function are in agreement with results in other literature. That is, it is the force factor Bl and the suspension compliance C_{ms} which has the largest effect. Interestingly, for this loudspeaker model, the suspension compliance C_{ms} does effect the THD only for very low frequencies. The voice coil inductance L_e, did on the other hand only have marginal effect in comparison to the other two. Studies have though shown that voice coil inductance does also have current dependency [Kli06]. What this means is that, if an IMD analyze is done one could see that voice coil inductance L_e is able to cause considerable IMD if a high frequency tone (> 1000 Hz) is played together with a low frequency one. Since this work did only focus on the loudspeaker working in the low frequency range, no simulation were done to verify this.

The observer based system is able to give better performance than the feed-forward system. It does though require considerable effort to tune the unscented Kalman filter to give optimal performance for the desired frequency range and cone excursion. As this is the case, one might guess if it wouldn't be more effective to put that effort in modelling the process noise directly in the model and utilize the augmented version of the UKF. This requires that an analysis of the difference between the loudspeaker model and the loudspeaker needs to be carried out. Also, since the UKF done in this work is a non-augmented version which assumes that the noise is additive, it does need to be changed to a augmented one.

There are other possibilities of modelling the moving coil loudspeaker as well. This includes Volterra series, which do allow for an easy model fitting. On the other hand, the complexity of such a model would be substantial if one would like to suppress more than two harmonics and as such it was not considered as a viable option. Also, any feedback other than the defined output could not be used to give correction to the model. Since this is the case, it is believed that a physical loudspeaker model should be preferred over a generic one.

6 Conclusions

As stated above, the controller is very sensitive to model errors. If the model error is too large it could render the controller destructive. On the other hand, if the loudspeaker model is accurate the controller does deliver excellent performance. It is possible to use an observer to compensate for small parameter variation but it does require some tuning of the unscented Kalman filter to get it to be optimal. Since some parameter variations are expected because of temperature dependency, aging and batch differences, a correct loudspeaker model should be striven for nonetheless. In other words, the UKF could help minimizing the effects of those parameters variations which could be difficult to model.

Since it was found that the observer based and feed-forward system had similar parameter sensitivity, it could be guessed that those parameters affect any feedback system the same. Obviously, one should try to model the parameters which has the greatest effect most accurately.

To be able to fully suppress the harmonic distortion it is necessary to have fast sampling frequency. That is, since the high frequency dynamics are depended on the sampling frequency, it can affect the controller ability to fully suppress the harmonic distortion. One important factor which could easily be overlooked is the step length of the Runge-Kutta algorithm in the controller. It was found that it could make the system unstable if chosen large enough. This parameter is though dependent on the sample frequency.

It should be noted that the loudspeakers physical properties do have their limits. That is the control signal power needed to make the loudspeaker linear for very low frequencies and large excursion can easily be enough to destroy the loudspeaker. When operating at low frequencies and large excursion the necessary power should also be deliverable from the amplifier. The loudspeaker model used in this work did though prove to be quite asymmetrical which put large stress on the control signal activity.

As stated before, the difference between the loudspeaker model and the loudspeaker were substantial. Since this work did not address the issue of measuring the loudspeaker, the results of why this difference occurs will remain inconclusive. In other words, if it would have been possible to include the model fitting in this work it would be possible to say how accurate it is in reality. One could though question if the accuracy of the terminal voltage, which was one decimal, were accurate enough. As well could one question the linearity of the amplifier, even though it was estimated that it would be negligible in comparison to the loudspeaker.

7 Future Work

Since this project was mostly focused on simulations, understandably it remains to verify and test most of the work described above. This chapter will try to list what is needed to be tested and suggest how the system can be extended to give a better performance. First, a few suggestions are given for how the loudspeaker model can be extended and secondly the same will be done for the observer.

7.1 Loudspeaker model work

The correctness of the loudspeaker model is the most important factor for the performance of the controller. Because of this, a few suggestion will be made of how the loudspeaker model can be more accurately fitted to a given loudspeaker.

First of all, it is possible to physically measure most of the nonlinear functions and constants. This does though involve some extensive lab work and disassembly of the loudspeaker.

Secondly, the loudspeaker model can be fitted to the given loudspeaker by measuring its external signals. This includes the terminal voltage, current, displacement and velocity. With these signals accurately known, the loudspeaker model can be fitted with common mathematic tools.

Thirdly, the loudspeaker model could possibly be fitted by a learning algorithm by measuring the harmonic distortion with a microphone. This of course, would require some good initial guess of the nonlinear functions but could possibly be done if the constants were known. This process obviously needs to be automated.

Besides fitting the existing model as well as possible, some extension could be made to better follow some other variations in the loudspeaker. As described in Chapter 3.1.2, this project omitted the thermal effects of the loudspeaker. As it is known that the heating of the voice coil does impact some of the parameters it is suggested that this should be modeled. The most significant parameter in this case is probably the voice coil resistance. Equation 7.1 shows how it can be extended to include the voice coil temperature T_v [BSA95].

$$R_e(T_v) = R_e(T_0)(1 + \alpha(T_v - T_0)) \tag{7.1}$$

where $\alpha = 4,33 \cdot 10^{-3} K^{-1}$ for copper. It could be difficult to measure the voice coil temperature but it should be possible to estimate it since the terminal voltage is known and the current either estimated or measured.

It is expected that the voice coil inductance L_e, has a weak nonlinear dependency on the current i [Kli06]. This could be incorporated into the loudspeaker model and investigated more detailed with IMD analyses.

The three major nonlinearities could also be extended to include the thermal effects. Most interestingly would probably be the suspension compliance C_{ms} since it is the one which varies the most with the temperature [Age07].

Another interesting extension could be to make the eddy current parameter R_2 and L_2 nonlinear in the model. It is known that they are weakly nonlinear for a common moving coil loudspeaker but if equipped with "shorting rings" it could be necessary to describe them more accurately. [DKOB04]

7.2 Observer work

One of the most obvious suggestion regarding the observer state estimation is that it should be implemented, verified to work and to be stable. Optimizing the UKF tuning parameters should also be of interest to get the most out of the observer. Other types of UKF, for example, the simplex UKF or the spherical UKF could be tried out if less computational effort is desired [Sim06].

A few extensions can made here as well. One of them is to make the UKF augmented, that is to assume that the noise is not additive. For example, the voice coil resistance can be multiplied with a constant which represent the process noise which is not currently modeled. This will allow the UKF to better estimate the states since the noise sources are more accurately implemented in the UKF. This, of course requires that it should be tested where the larges discrepancies are but a good initial guess could be the R_e and C_{ms} since it is known that they vary the most with time.

Another extension could be to make the unscented Kalman filter MIT-rule-based (MIT-AUKF) or make a master-slave UKF (MS-AUKF), which is based on two Kalman filters. Those two are both adaptive in the sense that they are able to change the covariance of the process noise $\mathbf{Q}$ over time. By doing this, slowly varying discrepancies could be compensated for more accurately. As described in Chapter 3.3.2, the covariance of the process noise $\mathbf{Q}$ is the parameter which affects the UKF the most in this case. To have it automatically adapt over time should therefore be of interest.[HSH09]

References

[Age07] F. Agerkvist. Modelling loudspeaker non-linearities. In *AES Conference:32nd International Conference: DSP For Loudspeakers*. Audio Engineering Society, September 2007.

[Ben93] J. E. Benson. *Theory and design of loudspeaker enclosures*. Synergetic Audio Concepts, 1993.

[BH09] M. R. Bai and C.-M. Huang. Expert diagnostic system for moving-coil loudspeakers using nonlinear modeling. *Acoustical Society of America Journal*, 125:819–830, 2009.

[BNS+98] M. A. Boer, A. G. J. Nijmeijer, H. Schurer, W. F. Druyvesteyn, C. H. Slump, and O. E. Hermann. Audibility of nonlinear distortion in loudspeakers. In *AES Convention:104*. Audio Engineering Society, May 1998.

[Bra92] M. Braun. *Differential Equations and Their Applications*. Springer-Verlag, New York, USA, December 1992.

[BSA95] G. Behler, U. Spätling, and T. Arimont. Measuring the loudspeaker's impedance during operation for the evaluation of the voice coil temperature. In *AES Convention:98*. Audio Engineering Society, February 1995.

[BX08] J. Blauert and N. Xiang. *Acoustics for Engineers*. Springer, 2008.

[DKOB04] M. Dodd, W. Klippel, and J. Oclee-Brown. Voice coil impedance as a function of frequency and displacement. In *AES Convention:117*. Audio Engineering Society, October 2004.

[GL00] T. Glad and L. Ljung. *Control Theory: Multivariable And Nonlinear Methods*. Tayler and Francis, London, Great Britain, 2000.

[HSH09] J. Han, Q. Song, and Y. He. Adaptive unscented kalman filter and its applications in nonlinear control. In V. M. Moreno and A. Pigazo, editors, *Kalman Filter: Recent Advances and Applications*, chapter 1, pages 1–24. INTECH, April 2009.

[Iai10] Iain. Loudspeaker cross section. `http://en.wikipedia.org/wiki/File:Speaker-cross-section.svg`, May 2010. Image.

[iec03] Sound system equipment - part 5: Loudspeakers. *IEC Publication 60268-5*, May 2003.

[Isi95] A. Isidori. *Nonlinear Control System: An Introduction*. Springer-Verlag, 1995.

[JS10] H. Jouni and S. Simo. Optimal filtering with kalman filters and smoothers – a manual for matlab toolbox ekf/ukf. `http://www.lce.hut.fi/research/mm/ekfukf/`, May 2010.

[JU04] S. J. Julier and J. K. Uhlmann. Unscented filtering and nonlinear estimation. In *IEEE*, volume 92(3), pages 401–422, 2004.

[KFI08] R. Kandepu, B. Foss, and L. Imsland. Applying the unscented kalman filter for nonlinear state estimation. *Journal of Process Control*, 18(7-8):753–768, 2008.

[Kli03] W. Klippel. Active compensation of transducer nonlinearities. In *AES Conference:23rd International Conference: Signal Processing in Audio Recording and Reproduction*, May 2003.

[Kli04] W. Klippel. Nonlinear modeling of the heat transfer in loudspeakers. *Journal of Audio Engineering Society*, 52:3–25, February 2004.

[Kli06] W. Klippel. Loudspeaker nonlinearities – causes, parameters, symptoms. *Journal of Audio Engineering Society*, 54:907–939, October 2006.

[Ped08] B. R. Pedersen. *Error Correction of Loudspeakers*. PhD thesis, Aalborg University, 2008.

[RLRV10] R. Ravaud, G. Lemarquand, T. Roussel, and Lemarquand V. Ranking of the nonlinearities of electrodynamic loudspeakers. *Archives of Acoustics*, 35(1):49–66, 2010.

[Sim06] D. Simon. *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*. Wiley-Interscience, June 2006.

[SL91] J.-J. E. Slotine and W. Li. *Applied nonlinear control*. Prentice-Hall, Englewood Cliffs, N.J., 1991.

[SSH96] H. Schurer, C. H. Slump, and O. E. Herrmann. Exact input-output linearization of an electrodynamical loudspeaker. In *AES Convention 101*. Audio Engineering Society, November 1996.

[Ter10] G. A. Terejanu. Unscented kalman filter tutorial. `http://www.acsu.buffalo.edu/~terejanu/tutorials.html`, May 2010.

[Thi71] A.N. Thiele. Loudspeakers in vented boxes, parts i and ii. *Journal of Audio Engineering Society*, 19:382–392, 471–483, 1971.

[win10] Windowing: Optimizing ffts using window functions. `http://zone.ni.com/devzone/cda/tut/p/id/4844`, May 2010.

Appendix

The measurement setup used to measure the loudspeaker is given below.

FFT		
Bandwith:	1,25	kHz
From:	0	kHz
To:	1,25	kHz
FFT lines:	1600	
Sample frequency:	3,2	kHz
Sample time:	1,28	s
Resolution:	781,3	mHz
wFFT window:	Hanning	
LP filter cut-off:	1,2	kHz

Table 7.1: Expermental setup

The instruments used to measure the loudspeaker are listed below.

Instruments		
Laser scanning head:	Polytec OFV 056 Vibrometer scanning head	SN: 6001881
Vibrometer controller:	Polytec vibrometer controller OFV 3001 S	Cert N: 5001550
Junction box:	Polytec Junction box PSV-Z-040	
Signal source:	EDIROL UA-101 24bit	TA NR: Re23
Oscilloscop:	Fluke 123 Scopemeter	
Amplifier:	NAD series 20 sterio amplifier 3020	TA NR: Po 9

Table 7.2: Lab instruments